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BOUNDARY HOMOGENIZATION AND REDUCTION OF DIMENSION IN THE KIRCHHOFF – LOVE PLATE

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I present a joint work with Dominique Blanchard (Université de Rouen, France) and Taras A. Mel'nyk (Kyiv Nat. Taras Shevchenko University, Ukraine)

We investigate the asymptotic behavior, as $\varepsilon \to 0$, of the Kirchhoff – Love equation satisfied by the transverse displacement U_ε of the middle surface $\Omega_\varepsilon^+ \cup \Omega_\varepsilon^-$ (contained in the (x_1, x_2)-coordinate plane) of a thin three-dimensional plate. The middle surface is composed of two domains. The first one Ω_ε^- is a thin strip with vanishing height h_ε (in direction x_2), as $\varepsilon \to 0$. The second one Ω_ε^+ is a comb with fine teeth having small cross section $\varepsilon\omega$ and constant height, ε-periodically distributed (in direction x_1) on the upper basis of the thin strip (see Figure 1). The middle surface is assumed clamped on the top of the teeth, with a free boundary elsewhere, and subjected to a transverse load.

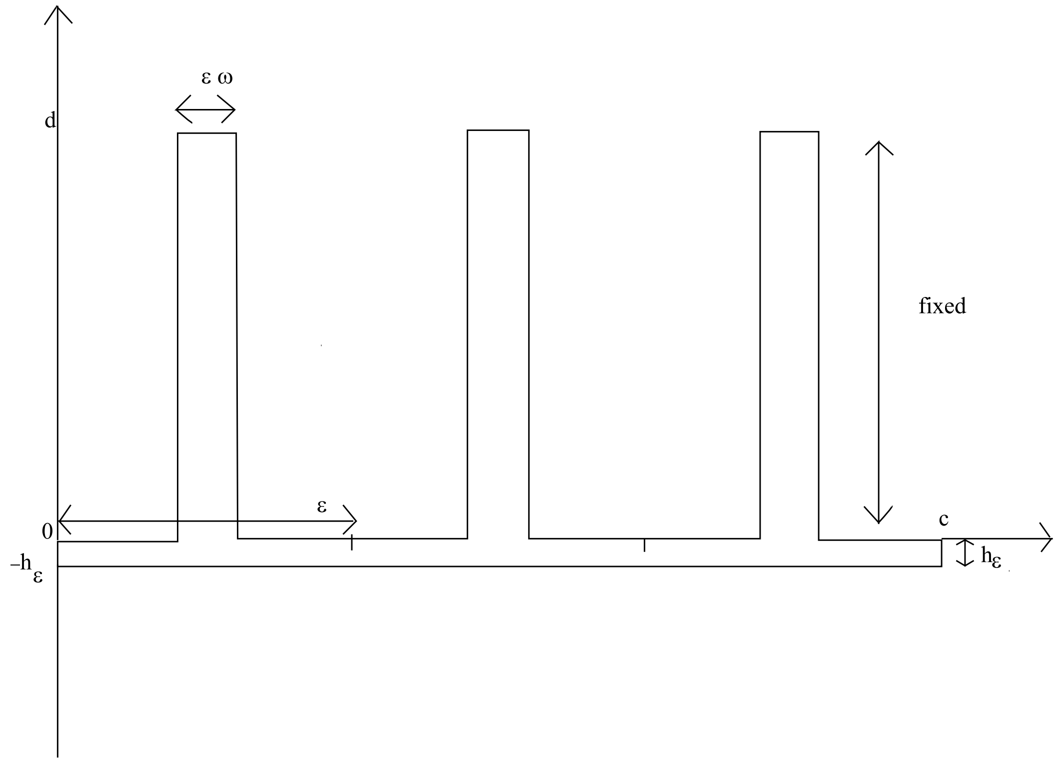

Fig. 1. The middle surface of our three-dimensional plate.

As $\varepsilon \to 0$, in the limit domain $\Omega^+ =]0, c[\times]0, d[$ of the comb, we obtain a continuum bending model of rods subjected to a force f, clamped on the upper side $\Gamma =]0, c[\times\{d\}$, and subjected on the lower side $\Sigma =]0, c[\times\{0\}$ to applied forces but without applied momentum. The forces on Σ depend on the limit density g of the transverse loads on the thin strip Ω_ε^-, and on the measure of the cross section ω of the reference tooth. The force f depends on the limit of the transverse loads on the teeth.

The limit displacement is independent of x_2 in the rescaled (with respect to h_ε) strip $\Omega^- =]0, c[\times] - 1, 0[$. The limit displacement meets a Dirichlet transmission condition between Ω^+ and Ω^-, if $h_\varepsilon \gg \varepsilon^4$, or if $h_\varepsilon \simeq \varepsilon^4$ and $\int_{-1}^0 g(x_1, x_2)dx_2 = 0$ a.e. in $]0, c[$. While, if the strip is thin enough and the transverse loads on the thin strip are strong enough, i.e. $h_\varepsilon \simeq \varepsilon^4$ and $\int_{-1}^0 g(x_1, x_2)dx_2 \neq 0$ in a subset of $]0, c[$ with positive measure, a discontinuity in the Dirichlet transmission condition appears. Roughly speaking, this means that the displacement in the strip oscillates between the teeth of Ω_ε^+ producing a limit average field different from that on the lower extremities of the teeth.

REFERENCES

1. *Allaire G.* Homogenization and Two-Scale Convergence // SIAM J. Math Anal. 1992. V. 23, N 6. P. 1482–1518.
2. *Amirat Y., Bodart O., De Maio U., Gaudiello A.* Asymptotic Approximation of the Solution of the Laplace Equation in a Domain with Highly Oscillating Boundary // SIAM J. Math Anal. 2004. V. 35, N 6. P. 1598–1616.
3. *Blanchard D., Carbone L., Gaudiello A.* Homogenization of a Monotone Problem in a Domain with Oscillating Boundary // M2AN Math. Model. Numer. Anal. 1999. V. 33, N 5. P. 1057–1070.
4. *Blanchard D., Gaudiello A.* Homogenization of Highly Oscillating Boundaries and Reduction of Dimension for a Monotone Problem // ESAIM Control Optim. Calc. Var. 2003. V. 9. P. 449–460.
5. *Blanchard D., Gaudiello A., Mossino J.* Highly Oscillating Boundaries and Reduction of Dimension: the Critical Case // Anal. Appl. (Singap.), to appear.
6. *Brizzi R., Chalot J. P.* Boundary Homogenization and Neumann Boundary Value Problem // Ricerche Mat. 1997. V. 46, N 2. P. 341–387.
7. *Gaudiello A., Zappale E.* Junction in a Thin Multidomain for a Fourth Order Problem // M ^{3} AS: Math. Models Methods Appl. Sci. 2006. V. 16, N 12. P. 1887–1918.
8. *Keller J. B., Nevard J.* Homogenization of Rough Boundary and Interfaces // SIAM J. Appl. Math. 1997. V. 57, N 6. P. 1660–1686.
9. *Mel'nyk T. A.* Homogenization of the Poisson Equations in a Thick Periodic Junction // Z. Anal. Anwendungen. 1999. V. 18, N 4. P. 953–975.
10. *Mel'nyk T. A., Nazarov S. A.* Asymptotics of the Neumann Spectral Problem Solution in a Domain of “Thick Comb” Type. // J. Math. Sci. 1997. V. 85, N 6. P. 2326–2346.
11. *Nguetseng G.* A General Convergence Result for a Functional Related to the Theory of Homogenization // SIAM J. Math Anal. 1989. V. 20, N 3. P. 608–623.
12. *Tartar L.* Cours Peccot, Collège de France (March 1977). Partially written in Murat F. H-Convergence. Séminaire d'analyse fonctionnelle et numérique de l'Université d'Alger. 1977-78. English translation in Mathematical Modelling of Composite Materials, Cherkaev A. and Kohn R. V. ed., Progress in Nonlinear Differential Equations and their Applications. Birkhäuser, Verlag, 1997. P. 21–44.