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COMPLEX PARTIAL DIFFERENTIAL MODEL EQUATIONS

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A model complex partial differential equation is an equation the partial differential operator of which is just the main part of a linear differential operator. Typical examples are the Cauchy – Riemann operator, the Bitsadze, the Laplace, the polyanalytic, the polyharmonic operator and any product of powers of the Cauchy – Riemann and the anti Cauchy – Riemann operators. The fundamental solutions to these operators are contained in a family of kernel functions leading to a hierarchy of higher order Pompeiu integral operators appearing in higher order Cauchy – Pompeiu representations. The basic Pompeiu operator and the simplest Cauchy – Pompeiu formula were fundamental tools in I. N. Vekua's treatment of the generalized Beltrami equation, in particular for developing his theory of generalized analytic functions. In the same way any complex higher order linear differential equation can be treated. Using potentials of the leading term of the differential operator given in form of the respective higher order Pompeiu operator the differential equation can be rewritten as a singular integral equation. This integral equation can under some ellipticity condition be treated by the Fredholm theory.

The Cauchy – Pompeiu representations are in general not proper for solving boundary value problems. Hence they need modification in order to be useful for solving boundary value problems. Such boundary value problems are of Schwarz, Dirichlet, Neumann, Robin type. For higher order equations however a variety of boundary value problems arise by combining these basic problems. Not all of them are well posed so that solvability conditions have to be determined before solutions can be looked for. A justification for posing boundary value problems is the aesthetics of the resulting representation formula of the solution. Explicit solutions can be only attained for particular domains. These are discs, half planes, quarter planes etc. Particular kernel functions appearing are the higher order Schwarz kernel, polyharmonic Green, Neumann, Robin functions and hybrid ones consisting of convolutions of lower order ones. There is thus a variety of such kernel functions and it becomes a combinatorial problem to determine all of them. This is the more as there are other polyharmonic Green – Almansi functions not given in an inductive way by a convolution process. The latter were used by I. N. Vekua in solving a Dirichlet boundary value problem for polyharmmonic functions. Using the proper higher order Pompeiu operator this Dirichlet problem can be solved for the related higher order Poisson equation. But as was just mentioned there is a variety of Dirichlet problems for this equation. Some of them are formulated and solved.