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GENERALIZED ANALYTIC FUNCTIONS IN FRACTIONAL SPACES AND SOME APPLICATIONS

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1^0. We consider the differential equation

$$\frac{\partial w}{\partial \overline{z}} + A(z)w + B(z)\overline{w} = 0, \quad \frac{\partial}{\partial \overline{z}} = \frac{1}{2}\left(\frac{\partial}{\partial x} + i\frac{\partial}{\partial y}\right), \tag{1}$$

in a bounded domain G of points $z = x + iy$.

The theory of I. N Vekua [1] of generalized analytic functions, which are the generalized (in some sense) solutions of the equation (1), where $A(z)$, $B(z) \in L_q(G)$, $q > 2$, have found many real targets of applications. A somewhat different approach to the theory is presented in [2]. A very wide class of an elliptic system of equations of a more general form reduced to the form (1).

The generalized in the sense of [1] solutions of equation (1) with coefficients $A(z)$ and $B(z)$ from Nikol'skii – Besov spaces $B^{\alpha}_{p,\theta}(G)$, where α, p, θ satisfy one of the conditions

$$\text{a) } 1 < p < 2, \quad \alpha = \frac{2}{p} - 1, \quad \theta = 1,$$

$$\text{b) } p \geq 2, \quad 0 < \alpha < 1, \quad 1 \leq \theta \leq \infty,$$

G is a domain with Lyapunov's boundary $\Gamma \in C^1_{\nu}$, $\alpha < \nu \leq 1$, are the generalized analytic functions [3]. The regular solutions of the equation (1) belong to $B^{1+\alpha}_{p,\theta}(G)$. In the case b) there occurs the imbedding $B^{\alpha}_{p,\theta}(G) \subset L_q(G)$ for some $q > 2$, that we have the case [1]. However, the assertion on the unconditional solvability of equation (1) in the fractional spaces $B^{1+\alpha}_{p,\theta}(G)$ is a new one. In the case a), for $1 < p < 2$, $\alpha = \frac{2}{p} - 1$, $B^{\alpha}_{p,1}(G)$ is not imbedded in $L_q(G)$ for any $q > 2$, but $B^{1+\alpha}_{p,1}(G) \subset C(\overline{G})$ [4]. Thus, this assertion introduces a new class of coefficients, not included in $L_q(G)$, $q > 2$, for which equation (1) always has a (regular) solution, which is continues (from $B^{1+\alpha}_{p,1}(G)$) in closed region $\overline{G}$. Moreover, in the case b) for $\alpha p = 2$ and $\theta = 1$, we have $B^{\frac{2}{p}}_{p,1}(G) \subset C(\overline{G})$, but $B^{\frac{2}{p}}_{p,1}(G) \not\subseteq C_{\beta}(\overline{G}), 0 < \beta \leq 1$, $B^{1+\frac{2}{p}}_{p,1}(G) \subset C^1(\overline{G})$. Thus, it follows from this assertion that, for any continuous (not necessarily Holder continuous) $A(z)$, $B(z) \in B^{\frac{2}{p}}_{p,1}(G)$, the equation (1) has a solution in the classical sense. This is a new property of equation (1) (in general for elliptic equations) has been proved for any Holder continuous coefficients.

2^0. The Cauchy type integral

$$\Phi(z) = \frac{1}{2\pi i}\int\limits_{\Gamma} \frac{f(\tau)d\tau}{\tau - z}, \quad z \in G,$$

with arbitrary density $f(t) \in B^{\frac{1}{p}}_{p,1}(\Gamma) \subset C(\Gamma)$, $1 < p < 2$, as a function of z belongs to $B^{\frac{2}{p}}_{p,1}(G) \subset C(\overline{G})$ [3]. This result seems interesting because it is known that a Cauchy type integral with arbitrary continuous density in general need not be a continuous function in the closed domain.

3^0 . Consider the singular integral equation

$$a(t)f(t) + \frac{b(t)}{\pi i} \int\limits_{\Gamma} \frac{f(\tau)d\tau}{\tau - t} + (Kf)(t) = g(t), \quad t \in \Gamma, \tag{2}$$

where $a(t)$, $b(t)$, $g(t) \in B^{\frac{1}{p}}_{p,1}(\Gamma)$, $1 < p < 2$, are given functions, K is a compact operator in $B^{\frac{1}{p}}_{p,1}(\Gamma)$. Let $a^2(t) - b^2(t) \neq 0$ on Γ i. e., (2) is the elliptic equation.

The equation (1) is Fredholm one in $B^{\frac{1}{p}}_{p,1}(\Gamma) \subset C(\Gamma)$, $1 < p < 2$. It is possible to show for equation (2) the spaces of Fredholm's solvability in the cases, when ellipticity is violated at the finitely many points on Γ [5].

The analogous results are valid for the systems of the equations of type (2).

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