УДК 517.54

ON CAUCHY AND MORERA TYPE CRITERIONS FOR BOUNDEDNESS OF THE COEFFICIENT OF DISTORTION

© Mikhail Korobkov

korob@math.nsc.ru

Sobolev Institute of Mathematics, Novosibirsk, Russia

We give criteria for a mapping to have bounded distortion in terms of an integral estimate of the multiplicity function without any a priori assumption on the differential properties of the mapping. In this talk we extend some results of [1].

Let Δ be a domain in $\mathbb{R}^n$, $n = 2, 3, \ldots$. Recall that a continuous mapping $f = (f_1, \ldots, f_n) : \Delta \to \mathbb{R}^n$ is *a mapping with bounded distortion* [2] if the following conditions are fulfilled:

(i) $f \in W^{1,n}_{loc}(\Delta)$;

(ii) The Jacobian $J(f,x) = \det\bigl(\frac{\partial f_k}{\partial x_l}\bigr) \ge 0$ almost everywhere (a. e.) in Δ;

(iii) There exists a constant $K \ge 1$ such that $|f'(x)|^n \le K n^{n/2} J(f,x)$ a. e. in Δ, where $|f'(x)| = \bigl(\sum_{k,l=1}^{n} (\frac{\partial f_k}{\partial x_l})^2\bigr)^{1/2}$ is the Hilbert norm of the derivative $f'(x)$. The least constant K is called the *distortion coefficient* (*dilatation*) of f [2].

Denote the differential form $f_k\, dx_1 \wedge \cdots \wedge \widehat{dx_l} \wedge \cdots \wedge dx_n$ by ω_{kl}. Given a ball $B = B(x,r) \subset \Delta$, consider the numerical $(n \times n)$-matrix

$$\Omega(B) = \Bigl(\int\limits_{\partial B} \omega_{kl},\ 1 \le k, l \le n\Bigr).$$

Endow the space $\mathcal{M}_n$ of all $(n \times n)$-matrices with the Hilbert norm

$$|(a_{kl})| = \Bigl(\sum_{k,l} a_{kl}^2\Bigr)^{\frac{1}{2}}.$$

Theorem. *Let $f : \Delta \to \mathbb{R}^n$ be a continuous mapping of a domain $\Delta \subset \mathbb{R}^n$. Then $\exists K_0 > 1$ (K_0 doesn't depend on f) such that f is a mapping with distortion at most $K \in [1, K_0]$ if and only if the inequalities*

$$\Bigl(\frac{|\Omega(B)|}{|B|}\Bigr)^n \le n^{n/2} K^n \frac{\int_{\mathbb{R}^n} N(f|_B, y)\, dy}{|B|} < \infty;$$

$$\det \Omega(B) \ge 0$$

hold for every ball $B = B(x,r)$ such that $B(x,2r) \subset \Delta$.

Here we used the following notations. $|E|$ is the Lebesgue measure of a set E, $N(f|_E, \cdot)$ is the multiplicity function of the restriction $f|_E$, i. e., $N(f|_E, y) = \operatorname{card}(f^{-1}(y) \cap E)$.

Remark. Under the extra topological assumption that f is sense-preserving, above Theorem was proved in [1, Theorem 1']. Thus, Theorem 1 is a substantial strengthening of Theorem 1' of [1].

The author was supported by the Russian Foundation for Basic Research (Grant 05-01-00482-(a)) and the President of the Russian Federation (Grant MK-3778.2004.1)

REFERENCES

1. *Kopylov A. P., Korobkov M. V., Ponomarev S. P.* Stability in the Cauchy and Morera theorems for holomorphic functions and their spatial analogs // Sibirsk. Mat. Zh. 2003. V. 44, № 1. P. 120–131.
2. *Reshetnyak Yu. G.* Spase Mappings with Bounded Distortion. Providence, Amer. Math. Soc., 1989.