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TUTTE'S PROBLEM ON THE NUMBER OF MAPS ON RIEMANN SURFACE

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Let S be a closed Riemann surface. A graph G embedded into S is called a *map* or *dessins d'enfants* if any connected component of $S\backslash G$ is a disc. Topological and combinatorial background of the theory of maps was created by Tutte in a series of "Census"papers in 1962-1963. Later, in his famous Esquisse d'un programme (1984) Grothendick relied the investigation of maps with many problems in Complex Analysis, Combinatorial Theory, Number Theory, and Theory of Fuchsian groups. In particular, it turns that any map on Riemann surface is canonically associated with a meromorphic function having three critical values (Belyi function). In this case, Riemann surface is defined by an algebraic equation whose coefficients are algebraic numbers.

Two maps G and G' on Riemann surface S are *equivalent* if there is an orientation preserving homeomorphism of S sending G onto G'. A map is called *rooted* if one of its oriented edges is distinguished as a root. Isomorphisms between rooted maps take root into root. The main object of our consideration is the Tutte problem on the number of non-isomorphic maps of given genus with given number of edges х6ъ. Rooted version of this problem for genus [6] was solved by Tutte himself. A explicit formula for the number of rooted maps on the torus $(g = 1)$ was obtained by D. Arquès [1]. The generating functions for genus $g = 2$ and $g = 3$ cases were derived in [2]. In [4] a new method was suggested to calculate the number of conjugacy classes of subgroups in an arbitrary finitely generated group. As an application of this method we give the complete solution of Tutte's problem for the maps of prescribed genus and with given number of edges [5]. Earlier, this problem was solved only for the sphere [3]. As a further development of the method we suggest a new formula for the number of chiral pairs of maps (*twins*) with prescribed number of edges.

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