Международная конференция "Дифференциальные уравнения, теория функций и приложения", 2007, с. 499–500

УДК 517.581

ON THE GENERALIZED TRICOMI' FUNCTION

© N. Virchenko

lr@online.com.ua

National Technical University of Ukraine "KPI", Kyiv, Ukraine

Let us consider the following generalization (according to Wright) of the Tricomi' function:

$$\Psi^{\tau,\beta}(a;c;z)=\frac{1}{\Gamma(a)}\int_0^\infty t^{a-1}(1+t)^{c-a+1}\ {}_1\Psi_1\left[\begin{array}{c}(a,\beta)\\(a,\tau)\end{array}\middle|\ -zt^\tau\right]dt, \tag{1}$$

where $z\in\mathbf{C}$; $a,c\in\mathbf{C}$; $\tau,\beta\in\mathbf{R}$, $\tau>0$, $\beta-\tau>-1$, $\Gamma(a)$ is the Euler gamma-function [1], ${}_1\Psi_1$ is the Fox – Wright function [2]; as $\beta=\tau=1$ we have classical Tricomi function $\Psi(a;c;z)$ [1].

Some properties of (1) are studied, in particular, the differential, integral properties, and some applications are given [3].

Theorem 1. *For* $z\in\mathbf{C}$, *complex* $a,c\in\mathbf{C}$; $\tau,\ \beta\in\mathbf{R}$, $\tau>0$, $\beta-\tau>-1$ *there hold the next relations:*

$$\frac{d}{dz}\Psi^{\tau,\beta}(a;c;z)=-\frac{\Gamma(a+\beta)\Gamma(a+\beta-c-\tau+1)}{\Gamma(a)\Gamma(a-c+1)}\Psi^{\tau,\beta}(a+\beta;\ c+\tau;\ z),$$

$$(a>c-1,\ a>-\beta);$$

$$\frac{d^n}{dz^n}\Psi^{\tau,\beta}(a;c;z)=(-1)^n\frac{\Gamma(a+n\beta)\Gamma(a+n\beta-c-n\tau+1)}{\Gamma(a)\Gamma(a-c+1)}\Psi^{\tau,\beta}(a+n\beta;\ c+n\tau;\ z);$$

$$\frac{d}{dz}\left(z^a\Psi^{\tau,\beta}(a;c;z^\beta)\right)=a(a-c+1)z^{a-1}\ \Psi^{\tau,\beta}(a+1;\ c;\ z^\beta);$$

$$\frac{d}{dz}\left(z^{1-c}\Psi^{\tau,\beta}(a;c;z^{-\tau})\right)=(a-c+1)z^{-c}\ \Psi^{\tau,\beta}(a;\ c-1;\ z^{-\tau}).$$

Theorem 2. *If* $z\in\mathbf{C}$; $a,c\in\mathbf{C}$; $\tau,\beta\in\mathbf{R}$, $\tau>0$, $\tau-\beta<1$, $a\neq1$, $a\neq c$, *then the following relations are valid:*

$$\int_0^z t^{a-2}\Psi^{\tau,\beta}(a;\ c;\ t^\beta)dt=\frac{z^{a-1}}{(a-1)(a-c)}\ \Psi^{\tau,\beta}(a-1;\ c;\ z^\beta),$$

$$\int_0^z t^{-c-1}\Psi^{\tau,\beta}(a;\ c;\ t^{-\tau})dt=\frac{z^{-c}}{a-c}\ \Psi^{\tau,\beta}(a;\ c+1;\ z^{-\tau}),$$

$$\int_0^1 (1-t)^{-a-c}t^{a-1}\Psi^{\tau,\beta}(a;\ c;\ z^{\tau+\beta}(1-t)^{-\tau})dt=B(a;t-c)\ \Psi^{\tau,\beta}(a;\ a+c;\ z^{\tau+\beta}).$$

The PROOFS of these two theorems are straightforward.

Theorem 3. *As* $a, c \in \mathbf{C}$, $\mathrm{Re}\, a > -1$; $\tau, \beta \in \mathbf{R}$, $\tau - \beta < 1$, $\mathrm{Re}\, a > -\beta$ *the following functional relation is valid:*

$$\Gamma(a+1)\Gamma(a-c+2)\ \Psi^{\tau,\beta}(a+1;\ c;\ z) - \Gamma(a+1)\Gamma(a-c+1)\ \Psi^{\tau,\beta}(a;\ c;\ z) = z\beta\Gamma(a+\beta)\Gamma(a+\beta-c-\tau+1)\ \Psi^{\tau,\beta}(a+\beta;\ c+\tau;\ z).$$

The PROOF of this theorem is carrying out by the comparison of the coefficients at z^n.

The (τ, β)-generalized Tricomi function can be used for the generalization of Γ, B, ζ, $L_\nu^\alpha(z)$-functions etc.

Let us give some of them.

a) (τ, β)-generalized Γ-function:

$$_{\tau,\beta}\Gamma_a^c(\alpha; \gamma, \omega; b) = \int\limits_0^\infty t^{\alpha-1}\ e^{-t^\omega}\ \Psi^{\tau,\beta}(a;\ c;\ bt^{-\gamma})dt,$$

where $\mathrm{Re}\, c > \mathrm{Re}\, a > 0$; $\mathrm{Re}\, \alpha > 0$; $\tau \in \mathbf{R}, \tau > 0$; $\beta \in \mathbf{R}$; $\beta > 0$, $b > 0$; $\gamma \geq 1$; $\omega > 0$; $\Psi^{\tau,\beta}(a; c; z)$ is the function (1).

b) (τ, β)-generalized Laguerre's function:

$$_{\tau,\beta}L_\nu^\alpha(z) = \frac{sin\nu}{\pi} \int\limits_0^1 t^{-\nu-1}(1-t)^{\alpha+\nu}\ {}_1\Psi_1 \left[\begin{array}{c} (\alpha+1, \tau) \\ (\alpha+1, \beta) \end{array} \middle| \ zt^\tau \right] dt,$$

where $\mathrm{Re}\, \alpha \geq -1$, $\mathrm{Re}\, (\alpha + \nu) > -\frac{1}{\alpha}$, ν is not integer; $\tau, \beta \in \mathbf{R}$, $\tau - \beta < 1$, ${}_1\Psi_1$ is the Fox – Wright function.

REFERENCES

1. *Erdelyi A., Magnus W., Oberhettinger F., and Tricomi F. G.* Higher Transcendental Functions. McGraw-Hill, New York, 1953. V. 1.
2. *Kilbas A. A., Saigo M., Trujllo* On the generalized Wright function // Fract. Calc. Appl. Anal. 2002. V. 5, N 4. P. 437–460.
3. *Virchenko N. O.* Generalized special functions and their applications // Nauk. visti. Kyiv: KPI, 2006. N 4. P. 42–49.