DETERMINATION OF COEFFICIENT OF RIGHT SIDE OF PARABOLIC EQUATION

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Abstract: In this paper, the inverse problem for determination of the function not depending on one of space coordinates of semi-linear parabolic equation in the right hand side is investigated. The additional condition given in the integral form characterizes the specification of a change of the total amount of heat in one of the directions. Moreover, theorems which are related to existence, uniqueness and stability of solution of considered problem have been proved.

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Denote by D', bounded region of R^{n-1} and $D = D' \times (a,b) \subset R^n$, where a, b are any real numbers, $x' = (x_1,...,x_{n-1})$, $x = (x', x_n)$ are respectively any points of the regions D' and D and denote $Q' = D' \times (0,T]$, $Q = D \times (0,T]$, $S = \partial D \times [0,T]$, $0 < T = const$.

Let us consider the following problem to determinate the pair $\{f(x',t),\ u(x,t)\}$:

$$u_t - \Delta u = f(x',t)g(u), \quad (x,t) \in Q \tag{1}$$

$$u(x,0) = \varphi(x), \quad x \in \overline{D} = D \cup \partial D; \quad u(x,t) = \psi(x,t), \quad (x,t) \in S \tag{2}$$

$$\int_a^b u(x', x_n, t)dx_n = h(x',t), \quad (x',t) \in \overline{Q}' \tag{3}$$

where $u_t = \dfrac{\partial u}{\partial t}$, $u_{x_i} = \dfrac{\partial u}{\partial x_i}$, $\Delta u = \sum_{i=1}^{n} \dfrac{\partial^2 u}{\partial x_i^2}$ - is a Laplacian operator, $g(\cdot)$, $\varphi(\cdot)$, $\psi(\cdot)$, $h(\cdot)$ - are given functions.

As known that this kind of problems are not well –posed problems in the sense of Hadamard and have been studied for example in [1-9].

The structure of right-hand side alloqs us to apply the obtained recelts to inverse problems of determination of

1) coefficient of u_x(x,t) or u (x, t)

2) coefficient of right-hand side of reaction-diffusion type system (sec Remark)

If the functions $f(x',t)$ are given in the equation (1) then naturally, the condition (3) will not be given. Investigation of existence of the solution of the problem (1)-(2) is obvious as generally. For example it has been observed in [10-12], etc.

We shall assume the following assumptions related to the data of problem:

$1^0 \cdot g(\cdot) \in Lip_{(loc)}(R^1)$, $\quad |g(\cdot)| \geq m > 0$, m - real number;

$2^0 \cdot \varphi(\cdot) \in C^{2+\alpha}(\overline{D}), \psi(x,t) \in C^{2+\alpha,1+\alpha/2}(S)$, $\quad \varphi(x) = \psi(x,0)$, $\quad x \in \partial D$;

$3^0 \cdot h(x',t) \in C^{2+\alpha,1+\alpha/2}(\overline{Q'})$, $\quad h(x',0) = \int_a^b \varphi(x)\,dx_n$, $\quad x' \in \overline{D'}$, $\quad h(x',t) = \int_a^b \psi(x,t)\,dx_n$, $(x',t) \in \partial D' \times [0,T]$.

$$4^0 \cdot [\psi_t(x,0) - \Delta\varphi(x)] \cdot \int_a^b g(\varphi(x) dx_n) = \left[h_t(x',0) - \Delta h(x',0) - \psi_{x_n}(x',b,0) + \psi_{x_n}(x',a,0)\right] \times$$
$$\times\, g(\varphi(x)), \; x \in \partial D, \; x' \in \partial D'$$

Definitions of the spaces $C^{k+\alpha,(k+\alpha)/2}(\cdot)$, $\quad k = 0,1,2$, $\quad 0 < \alpha < 1$ and norms in these spaces are given in [10, p.16].

Definition 1. *If the following conditions are satisfied, the pair* $\{f(x',t), u(x,t)\}$ *is called the solution of problem (1)-(3): 1)* $f(x',t) \in C(\overline{Q'})$; *2)* $u(x,t) \in C^{2,1}(\overline{Q})$; *3) the functions* f *and* u *satisfy equalities (1)-(3).*

Let us denote $K_\alpha = \left\{(f,u) \middle| f(x',t) \in C^{\alpha,\alpha/2}(\overline{Q}'), \quad u(x,t) \in C^{1+\alpha,\alpha/2}(\overline{Q})\right\}$.

The problem of uniqueness of a solution is important in the coefficient inverse problems. Theorems on uniqueness of a solution guarantee the result of suitable physical experiments for this kind of problems. Moreover, if solution of inverse problem is searched in a compact set then also uniqueness theorems guarantee the stability of solution. For this reason, investigation of conditional stability for inverse problems that is evaluated of continuity module of inverse operator is important.

Theorem 1. *Assume that the conditions* $1^0 - 3^0$ *are satisfied. Then if the solution of the problem (1)-(3) exists and belongs to set* K_α*, this solution is unique and satisfies the following estimate:*

$$\|u-\overline{u}\|_0 + \|f-\bar{f}\|_0 \le M_1 \left[\|g-\overline{g}\|_0 + \|\varphi-\overline{\varphi}\|_2 + \|\psi-\overline{\psi}\|_{2,1} + \|h-\overline{h}\|_{2,1}\right], \quad (4)$$

where $\|\cdot\|_k = \|\cdot\|_{C^k}$*. The pair* $\{\bar{f}(x',t), \ \overline{u}(x,t)\}$ *is a solution of the problem (1)-(3) belonging to the set* K_α *with the data* $\overline{g}(\cdot), \ \overline{\varphi}(\cdot), \ \overline{\psi}(\cdot), \ \overline{h}(\cdot)$, *satisfying the conditions* 1^0*-*3^0*. The number* $M_1 > 0$ *depends on the set* K_α *and the data (especially, we will denote the constants which depend on the set of* K_α *and the data of problem by* M_i *and denote the constants depending only on the data of problem by* N_i*).*

Proof. Integrating both parts of equation (1) by variable x_n on interval (a,b) and taking into account conditions of theorem 1, for function $f(x',t)$ we get:

$$f(x',t) = \left[h_t(x',t) - \Delta h(x',t) - u_{x_n}(x',b,t) + u_{x_n}(x',a,t)\right] \Big/ \int_a^b g(u)dx_n, \quad (x',t) \in \overline{Q}', \quad (5)$$

Let us define the function [11, p 87]

$$p(x,t) \in C^{2+\alpha,1+\alpha/2}(\overline{Q}), \quad p(x,0) = \varphi(x), \ x \in \overline{D}, \quad p(x,t) = \psi(x,t), \ (x,t) \in S, \quad (6)$$

We put

$$z(x,t)=u(x,t)-\bar{u}(x,t),\ \lambda(x',t)=f(x',t)-\bar{f}(x',t),\ \delta_1(u)=g(u)-\bar{g}(u),$$

$$\delta_2(x)=\varphi(x)-\bar{\varphi}(x),\ \delta_3(x,t)=\psi(x,t)-\bar{\psi}(x,t),$$

$$\delta_4(x',t)=h(x',t)-\bar{h}(x',t)\ \delta_5(x,t)=p(x,t)-\bar{p}(x,t).$$

It is seen easily that pair of the functions $\{\lambda(x',t),\upsilon(x,t)=z(x,t)-\delta_5(x,t)\}$ satisfies the following system:

$$\upsilon_t-\Delta\upsilon=\lambda(x',t)g(u)+F(x,t),\ \ (x,t)\in Q, \tag{7}$$

$$\upsilon(x,0)=0,\ \ x\in\bar{D};\ \ \upsilon(x,t)=0,\ \ (x,t)\in S, \tag{8}$$

$$\lambda(x',t)=\left[\delta_{4t}(x',t)-\Delta\delta_4(x',t)-z_{x_n}(x',b,t)+z_{x_n}(x',a,t)\right]\Big/\int_a^b g(u)dx_n+H(x',t),$$

$$(x',t)\in\bar{Q}', \tag{9}$$

where $F(x,t)=\bar{f}(x',t)\ [g(u)-\bar{g}(\bar{u})]-\delta_{5t}(x,t)+\Delta\delta_5(x,t),\ \ H(x',t)=\{[\bar{h}_t(x',t)-$

$$-\Delta\bar{h}(x',t)-\bar{u}_{x_n}(x',b,t)+\bar{u}_{x_n}(x',a,t)]\int_a^b[\bar{g}(\bar{u})-g(u)]dx_n\Big\}\Big/\left[\int_a^b g(u)dx_n\int_a^b\bar{g}(\bar{u})dx_n\right].$$

When the conditions of the theorem are satisfied, the functions in the right hand side of the equation (7) belong to Holder class. Therefore, classical solution of the problem (7)-(8) exists and it has the following representation [10, p.468]:

$$\upsilon(x,t)=\int_0^t\int_D G(x,t;\xi,\tau)[\lambda(\xi',\tau)g(u)+F(\xi,\tau)]d\xi\,d\tau \tag{10}$$

where $\xi'=(\xi_1,...,\xi_{n-1}),\ \xi=(\xi',\xi_n),\ d\xi=d\xi_1...d\xi_n,\ G(\cdot)$ is a Green function of the problem (7), (8) and satisfies the following inequalities [10, chapter IV]:

$$|G(x,t;\xi,\tau)|\le N_1(t-\tau)^{-n/2}\exp\left(-N_2|x-\xi|^2/(t-\tau)\right),$$

$$\int_D\left|D_x^k G(x,t;\xi,\tau)\right|d\xi\le N_3(t-\tau)^{-(k-\alpha)/2},\quad k=0,1,2, \tag{11}$$

and $D_x^k\cdot$ represents the derivatives of the k-th order with respect to the variables $x_i\left(i=\overline{1,n}\right)$.

If we consider $\upsilon(x,t)=z(x,t)-\delta_5(x,t)$, then we get from (10) that

$$z(x,t) = \delta_5(x,t) + \int_0^t \int_D G(x,t;\xi,\tau)[\lambda(\xi',\tau)g(u) + F(\xi,\tau)]d\xi d\tau \quad (12)$$

Let $\chi \equiv \|u - \bar{u}\|_0 + \|f - \bar{f}\|_0$.

Using the conditions of the theorem, the definition of the set K_α and the inequality (11) from (12) and (9) we get

$$|z(x,t)| \le M_2\left[\|\delta_1\|_0 + \|\delta_5\|_{2,1}\right] + M_3\chi t, \quad (x,t) \in \overline{Q} \quad (13)$$

$$|\lambda(x',t)| \le M_4\left[\|\delta_1\|_0 + \|\delta_4\|_{2,1} + \|\delta_5\|_{2,1}\right] + M_5\chi\, t^{(1+\alpha)/2}, \quad (x',t) \in \overline{Q}' \quad (14)$$

The inequalities (13), (14) are satisfied for all $(x,t) \in \overline{Q}$. For this reason, these inequalities will be satisfied for maximal values of the expression in the right hand side. Therefore

$$\chi \le M_6\left[\|\delta_1\|_0 + \|\delta_4\|_{2,1} + \|\delta_5\|_{2,1}\right] + M_7\chi\, t^{(1+\alpha)/2} \quad (15)$$

Let us choose the number $T_1(0 < T_1 \le T)$ such that $M_7 T_1^{(1+\alpha)/2} < 1$. Then we get from (15) that the estimate (4) is satisfied for the solution of problem (1)-(3) for $(x,t) \in \overline{D} \times [0,T_1]$.

If we examine the problem (1)-(3) in the regions $\overline{D} \times [T_1, 2T_1]$, $\overline{D} \times [2T_1, 3T_1]$ and etc. after the finite number of steps we get that estimate (4) is satisfied in region $\overline{D} \times [0,T]$.

Putting

$$g(u) = \bar{g}(u), \quad \varphi(x) = \bar{\varphi}(x), \quad \psi(x,t) = \bar{\psi}(x,t), \quad h(x',t) = \bar{h}(x',t),$$

in the estimate (4), we obtain the uniqueness of the solution of the problem (1)-(3).

Existence of the solution of the problem (1)-(3) in the of concept given in Definition 1 is proved by the successive approximations. The algorithm to find the pair of functions $\{f^{(s)}(x',t), u^{(s)}(x,t)\}$, $s = 1,2,...$ is as follows

$$u_t^{(s+1)} - \Delta u^{(s+1)} = f^{(s)}(x',t)g(u^{(s)}), \quad (x,t) \in Q \quad (16)$$

$$u^{(s+1)}(x,0) = \varphi(x), \ x \in \overline{D}; \quad u^{(s+1)}(x,t) = \psi(x,t), \quad (x,t) \in S \quad (17)$$

$$f^{(s+1)}(x',t)=\left[h_t(x',t)-\Delta h(x',t)-u_{x_n}^{(s+1)}(x',b,t)+u_{x_n}^{(s+1)}(x',a,t)\right]\Big/\int_a^b g\left(u^{(s+1)}\right)dx_n\,,$$

$$(x',t)\in\overline{D}' \qquad (18)$$

Theorem 2. *Let the conditions* 1^0-4^0 *be satisfied, where* $\partial D\in C^{2+\alpha}$. *Then the problem (1)-(3) has at least one solution in the sense of Definition 1.*

Proof. It is easu to show that if we choose $u^{(0)}(x,t)\in C^{2+\alpha,1+\alpha/2}\left(\overline{Q}\right)$, $f^{(0)}(x',t)\in C^{\alpha,\alpha/2}\left(\overline{Q}'\right)$, then under the conditions of Theorem 2 $u^{(1)}(x,t)\in C^{2+\alpha,1+\alpha/2}\left(\overline{Q}\right)$ [10, p.364]. Then, from (18) under the conditions of Theorem 2, it follows, that $f^{(1)}(x',t)\in C^{\alpha,\alpha/2}\left(\overline{Q}'\right)$. Consequently, we can assert, that functions $f^{(s)}(x',t)$ and $u^{(s)}(x,t)$ obtained from system (16)-(18) for $s=0,1,2,...$ belong to the functional spaces $C^{\alpha,\alpha/2}\left(\overline{Q}'\right)$ and $C^{2+\alpha,\,1+\alpha/2}\left(\overline{Q}\right)$, respectively. Let's show the uniform boundedness of sequences $\left\{f^{(s)}(x',t)\right\}$, $\left\{D_x^k u^{(s)}(x,t)\right\}$, $k=0,1,2$.

Using functions $p(x,t)$, defined in (6), and representation of solution by Green function [4, p.468] we get the following expression for the solution $u^{(s+1)}(x,t)$ of (16) and (17).

$$u^{(s+1)}(x,t)=p(x,t)+\int_0^t\int_D G(x,t,\xi,\tau)\left[f^{(s)}(\xi',\tau)g\left(u^{(s)}\right)+\Delta p-p_\tau\right]d\xi\,d\tau\,. \quad (19)$$

Similarly, to the proof of Theorem 1, using the inequality (11) and the conditions of Theorem 2, we obtain:

$$\left|D_x^k u^{(s+1)}(x,t)\right|\le N_4\|p\|_{2,1}+N_5t^{(2+\alpha-k)/2}\left|f^{(s)}(x',t)\right|,\qquad k=0,1,2,\quad (x,t)\in\overline{Q},$$

$$\left|f^{(s+1)}(x',t)\right|\le N_6\left[\|h\|_{2,1}+\|p\|_{2,1}\right]+N_7t^{(1+\alpha)/2}\left|f^{(s)}(x',t)\right|,\qquad (x,t)\in\overline{Q}',$$

or

$$\gamma^{(s+1)}\le N_8\left[\|h\|_{2,1}+\|p\|_{2,1}\right]+N_9t^{\alpha/2}\gamma^{(s)}$$

where $\gamma^{(s)}\equiv\sum_{k=0}^{2}\left\|D_x^k u^{(s)}\right\|_0+\left\|f^{(s)}\right\|_0$.

From the last inequality:

$$\gamma^{(s+1)} \le N_8 \left[\|h\|_{2,1} + \|p\|_{2,1}\right]\left(1-q^s\right)/(1-q) + q^s\gamma^{(0)}, \qquad q = N_9 t^{\alpha/2}$$

Let us choose the number $T_2\left(0 < T_2 \le T\right)$ such that $N_9 T_2^{\alpha/2} < 1$. Then we get that the sequences $\left\{f^{(s)}\right\}$, $\left\{D_x^k u^{(s)}\right\}$, $k = 0,1,2,...$ are uniformly (by norm C) bounded for $(x,t) \in \overline{D} \times [0, T_2]$.

As in the proof of Theorem 1, it is shown that the sequences $\left\{f^{(s)}\right\}$, $\left\{D_x^k u^{(s)}\right\}$, $k = 0,1,2$ are uniformly bounded for all $t \in [0, T]$.

Equicontinuity of the sequences $\left\{D_x^k u^{(s)}\right\}$, $k = 0,1,2$ is obtained from the inequalities

$$\left|D_x^k u^{(s+1)}(x,t) - D_x^k u^{(s+1)}(\bar{x},\bar{t})\right| \le \left|D_x^k u^{(s+1)}(x,t) - D_x^k u^{(s+1)}(\bar{x},t)\right| + \left|D_x^k u^{(s+1)}(\bar{x},t) - \right.$$

$$\left. - D_x^k u^{(s+1)}(\bar{x},\bar{t})\right| \le \left|D_x^k p(x,t) - D_x^k p(\bar{x},t)\right| + \left|D_x^k p(\bar{x},t) - D_x^k p(\bar{x},\bar{t})\right| +$$

$$+ \int_0^t \int_D \left|D_x^k G(x,t;\xi,\tau) - D_x^k G(\bar{x},t;\xi,\tau)\right| \cdot \left|F^{(s)}(\xi,\tau)\right| d\xi\, d\tau + \int_0^{\bar{t}} \int_D \left|D_x^k G(\bar{x},t;\xi,\tau) - \right.$$

$$\left. - D_x^k G(\bar{x},\bar{t};\xi,\tau)\right| \left|F^{(s)}(\xi,\tau)\right| d\xi\, d\tau + \int_{\bar{t}}^t \int_D \left|D_x^k G(\bar{x},t;\xi,\tau)\right| \left|F^{(s)}(\xi,\tau)\right| d\xi\, d\tau$$

(where $F^{(s)}(x,t) = f^{(s)}(x',t) g\left(u^{(s)}\right) + \Delta p - p_t$). Using the uniform boundedness of the sequence $\left\{f^{(s)}(x,t)\right\}$, $\left\{\boldsymbol{D}_x^k \boldsymbol{u}^{(s)}\right\}$, k=0,1,2 the continuity and boundedness of data, inequality (11) and the inequalities [10, p.469]

$$\left|D_x^k G(x,t;\xi,\tau) - D_{\bar{x}}^k G(\bar{x},t;\xi,\tau)\right| \le$$

$$\le N_{10} |x - \bar{x}|^\alpha (t-\tau)^{-(n+2+\alpha)/2} \exp\left(-N_{11} |x-\xi|^2 / (t-\tau)\right);$$

$$\left|D_x^k G(x,t;\xi,\tau) - D_x^k G(x,\bar{t};\xi,\tau)\right| \le \left\{D_x^k u^{(s)}\right\}, \; k = 0,1,2$$

$$N_{12} |t - \bar{t}|^{(2+\alpha-k)/2} (\bar{t} - \tau)^{-(n+2+\alpha)/2} \exp\left(-N_{13} |x-\xi|^2 / (t-\tau)\right).$$

Using uniform boundedness and equicontinuity of the sequence $\left\{D_x^k u^{(s)}\right\}$, $k = 0,1,2$ and continuity and boundedness of data, the equicontinuity of the sequence $\left\{f^{(s)}(x',t)\right\}$ is easily obtained from the following inequality:

$$\left|f^{(s)}(x',t)-f^{(s)}(\bar{x}',\bar{t})\right|\le\left|f^{(s)}(x',t)-f^{(s)}(\bar{x}',t)\right|+\left|f^{(s)}(\bar{x}',t)-f^{(s)}(\bar{x}',\bar{t})\right|\le$$

$$\le\Big[\left|h_t(x',t)-h_t(\bar{x}',t)\right|+\left|\Delta h(x',t)-\Delta h(\bar{x}',t)\right|+\left|u_{x_n}^{(s+1)}(x',b,t)-u_{x_n}^{(s+1)}(\bar{x}',b,t)\right|+$$

$$+\left|u_{x_n}^{(s+1)}(x',a,t)-u_{x_n}^{(s+1)}(\bar{x}',a,t)\right|\Big]/\left|\int_a^b g\left(u^{(s+1)}(x',x_n,t)\right)dx_n\right|+\Big\{\Big|h_t(\bar{x}',t)-\Delta h(\bar{x}',t)-$$

$$-u_{x_n}^{(s+1)}(\bar{x}',a,t)+u_{x_n}^{(s+1)}(\bar{x}',a,t)\Big|\left|\int_a^b\left[g\left(u^{(s+1)}(x',x_n,t)\right)-g\left(u^{(s+1)}(\bar{x}',x_n,t)\right)\right]dx_n\right|\Big\}/$$

$$/\int_a^b g\left(u^{(s+1)}(x',x_n,t)\right)dx_n\int_a^b g\left(u^{(s+1)}(\bar{x}',x_n,t)\right)dx_n\Bigg|+$$

$$+\Big[\left|h_t(\bar{x}',t)-h_t(\bar{x}',\bar{t})\right|+\left|\Delta h(\bar{x}',t)-\Delta h(\bar{x}',\bar{t})\right|+\left|u_{x_n}^{(s+1)}(\bar{x}',b,t)-u_{x_n}^{(s+1)}(\bar{x}',b,\bar{t})\right|$$

$$+\left|u_{x_n}^{(s+1)}(\bar{x}',a,t)-u_{x_n}^{(s+1)}(\bar{x}',a,\bar{t})\right|\Big]/\left|\int_a^b g\left(u^{(s+1)}(\bar{x}',x_n,t)\right)dx_n\right|+$$

$$+\Big\{\Big|h_t(\bar{x}',\bar{t})-\Delta h(\bar{x}',\bar{t})-u_{x_n}^{(s+1)}(\bar{x}',b,\bar{t})+u_{x_n}^{(s+1)}(\bar{x}',a,\bar{t})\Big|\Big|\int_a^b\Big[g\left(u^{(s+1)}(\bar{x}',x_n,t)\right)-$$

$$-g\left(u^{(s+1)}(\bar{x}',x_n,\bar{t})\right)\Big]dx_n\Big|\Big\}/\left|\int_a^b g\left(u^{(s+1)}(\bar{x}',x_n,t)\right)dx_n\cdot\int_a^b g\left(u^{(s+1)}(\bar{x}',x_n,\bar{t})\right)dx_n\right|.$$

Equicontinuity and uniform boundedness of the sequence $\{u_t^{(s)}\}$ is obtained from (16).

We get from Arzela Theorem [11, p.84] that there exist convergent subsequences of sequences of functions $\{u_t^{(s)}\}$, $\{D_x^k u^{(s)}\}$, $k=0,1,2$, $\{f^{(s)}\}$. Denote their limits by $\{u_t^*\}$, $\{D_x^k u^*\}$, $k=0,1,2$, $\{f^*\}$, respectively. Therefore convergent subsequences to the functions $u^*(x,t)\in C^{2,1}(\overline{Q})$, $f^*(x',t)\in C(\overline{Q'})$ exist.

Then if we pass to the limit in the expressions (16)-(18) when $s\to\infty$, we can easily show that the pair $\{f^*(x',t),\ u^*(x,t)\}$ satisfies the conditions (1)-(3).

So we have shown the existence of solution of the problem (1)-(3) in the sense of Definition1.

Remark. Analogiously to the problem (1)-(3), the inverse problems of determination of the functions $\{f_k(x',t), u_k(x,t), k=1,\overline{m}\}$ ($\{b_k(x',t), u_k(x,t), k=1,\overline{m}\}$ or $\{c_k(x',t), u_k(x,t), k=1,\overline{m}\}$) from the system of parabolic equations

$$u_{kt} - \Delta u_k + b_k(x',t)\sum_{i=1}^{n} u_{kx_i} + C_k(x',t)u_k = f_k(x',t)g(u_1,.....u_m),(x,t) \in Q$$

$$u_k(x,0) = \varphi_k(x),\ x \in \overline{D},\ u_k(x,t) = \psi_k(x,t), \quad (x,t) \in S$$

$$\int_a^b u_k(x',x_n,t)dx_n = h_k(x',t), \quad (x',t) \in \overline{Q'}$$

can be considered.

Results similar to those obtained above are obtained for this problem.

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