International Conference "Inverse and Ill-Posed Problems of Mathematical Physics",
dedicated to Professor M.M. Lavrent'ev on occasion of his 75-th birthday, August 20-25, 2007, Novosibirsk, Russia

Stability estimates for the Jacobi inverse eigenvalue problem

L. Borcea*, V. Druskin, L. Knizhnerman*****

* Computational and Applied Mathematics,
MS 134, Rice University, Main St., 6100,
TX 77005-1892 Houston, USA
E-mail: borcea@caam.rice.ru

** Schlumberger-Doll Research,
Hampshire St., 1,
MA 02139 Cambridge, USA
E-mail: druskin1@slb.com

*** Central Geophysical Expedition,
Narodnogo Opolcheniya St., 38-3,
123298 Moscow, Russia
E-mail: mmd@cge.ru

We present different stability estimates for the Jacobi inverse eigenvalue problem. First, we give upper bounds expressed in terms of quadrature data and not having weights in denominators. The technique of orthonormal polynomials and integral representation of Hankel determinants is used. Our bounds exhibit only polynomial growth in the problem's dimension (see [4]). It has been shown that the approach, exploiting integral representation of Hankel determinants, implies at least quantitative improvement of earlier results and generalizes them (see [5]). It has also been demonstrated that a particular implementation of the Hankel determinant approach gives an estimate being unimprovable up to a coefficient; the corresponding example involves quadrature data with a small but not too small weight. It follows that polynomial increase of a general case upper bound in terms of the dimension is unavoidable (see [5]). Second, we derive a differential formula for perturbations which allows us to effectively handle the special case of a spectral measure with approximately equal weights (see [2]). This special case is closely related to the inverse discrete Sturm-Liouville problem (see [1]), considered in combination with the "optimal finite difference grids" technique [3].

References

[1] L. Borcea, V. Druskin, L. Knizhnerman, On the continuum limit of a discrete inverse spectral problem on optimal finite difference grids, Comm. Pure Appl. Math. 58 (9) (2005), pp. 1231-1279.
[2] V. Druskin, L. Borcea, L. Knizhnerman, On the sensitivity of Lanczos recursions to the spectrum, Linear Algebra Appl., 396 (2005), pp. 103-125.
[3] V. Druskin and L. Knizhnerman, Gaussian spectral rules for the three-point second differences: I. A two-point positive definite problem in a semi-infinite domain, SIAM J. Numer. Anal., 37 (1999), No. 2, pp. 403-422.
[4] L. Knizhnerman, Stability estimates on the Jacobi and unitary Hessenberg inverse eigenvalue problems, SIAM J. Matrix Anal. Appl., 26 (2004), No. 1, pp. 154-169.
[5] L. Knizhnerman, Sensitivity of the Lanczos recurrence to Gaussian quadrature data: how much malignant can small weights be?, submitted.