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Coefficient inverse extremum problems for stationary models of heat and mass transfer

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Coefficient inverse problems for stationary models of heat and mass transfer are studied. These problems consist of finding unknown coefficients which enter the models under consideration using an additional information about the solution. The models consist of the Navier-Stokes equations and the convection-diffusion equations for the temperature and for the substance concentration that are nonlinearly related via buoyancy in the Boussinesq approximation and via convective heat and mass transfer. The inverse problems are stated as the minimization of certain cost functional at weak solutions to the original boundary value problem.

The solvability of these problems is proved, and optimality systems describing necessary optimality conditions are derived and analyzed. Our attention is focused on the analysis of the local uniqueness and stability of solutions to the coefficient inverse extremum problems. This analysis is rather difficult to perform, because the coefficient inverse problems are doubly nonlinear. We mean the nonlinearity in original model of heat and mass transfer and the nonlinearity associated with the unknown coefficients involved in these models. Nevertheless, the structure of the differential equations is such that a detailed analysis of the optimality systems derived produces sufficient conditions on the input data that ensure the local uniqueness and stability of solutions to particular coefficient inverse problems.

The local uniqueness conditions are rather complicated. To simplify them, we introduce analogues of dimensionless parameters widely used in fluid dynamics, namely, the Reynolds number and the temperature and diffusion Rayleigh and Prandtl numbers. In terms of these parameters, the uniqueness conditions can be written in a relatively simple form and are similar to those for the coefficient inverse problems for the stationary linear convection–diffusion–reaction equation. Detailed discussion of some results for models of heat convection, mass transfer and heat and mass transfer can be found in [1-4].

References

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