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The correct flow chart for numerical solving of an inverse problem by the optimization method

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We were forced to discuss this problem because it took part in many conferences devoted to inverse problems and their numerical solution.

The main point of this problem is the correct choice of the flow chart for numerical solving of an inverse problem.

Very often researchers choose the following flow chart: for example, for the reconstruction of unknown coefficient $q(x)$

- researchers have the direct problem $L_q u = 0$ (L is the operator of the direct problem) and the additional information on the direct problem solution u;
- they use the residual functional $J[q]$;
- and obtain the conjugate problem $L_q^* \psi = 0$;
- using solutions of direct u and conjugate ψ problems, researchers obtain the residual functional gradient $J'[q] = A(u, \psi)$ (A is the operator acting on the functions u and ψ);

after that for the numerical solving of the inverse problem researchers

- reduce the direct problem $L_q u = 0$ to the problem $\Lambda_p v = 0$ (Λ_p is the operator of the numerical solving of the direct problem, and the functions v and p are the approximations of the functions u and q respectively;
- use the residual functional $\Phi[p]$ which approximates the residual functional $J[q]$;
- reduce the conjugate problem $L_q^* \psi = 0$ to the problem $\tilde{\Lambda}_p \phi = 0$, ($\tilde{\Lambda}_p$ is the operator of the numerical solving of the conjugate problem, and the function ϕ is the approximation of the function ψ;
- obtain the relation $B(v, \phi)$ which approximates the residual functional gradient $J'[q] = A(u, \psi)$;

after that they use some kind of gradient methods for minimization.

Why is this flow chart wrong?

First, the equality $\tilde{\Lambda}_p v = \Lambda_p^* v$ is not necessarily true.
Second, the relation $B(v, \phi)$ can not be the gradient for the residual functional $\Phi[p]$.

In our opinion the correct flow chart must be as follows: for reconstruction of unknown coefficient q it is necessary that researchers

- reduce the direct problem $L_q u = 0$ to the problem $\Lambda_p\, v = 0$ which will be solved on computer;
- use the residual functional $\Phi[p]$;
- obtain the conjugate problem $\Lambda_p^* \phi = 0$;
- using the solutions of direct v and conjugate ϕ problems, obtain the residual functional gradient $\Phi'[p]$;

after that researchers can solve the minimization problem.

Why do researchers often choose the first flow chart?

Probably, the justification is the following reasons:

1. the operators $\tilde{\Lambda}_p$ and B approximate the operators L and A with some accuracy;
2. the additional information always has errors of measurements and calculations; it is the reason to assume that the calculation errors of the conjugate problem solution and of the residual functional gradient influence a little on the minimization and the inverse problem solution if to choose the first flow chart;
3. the first flow chart is more simple than the second one mathematically.

In our report we want to show that these reasons are wrong, and, consequently, the inaccuracy of the first flow chart will be shown.

We want to show it on the example of the numerical solving of the inverse problem of reconstruction of the unknown coefficient $q(x)$ from the following conditions:

$$u_{tt}(x,t) = u_{xx}(x,t) - q(x)u(x,t), \qquad (x,t) \in D = \{x \in (0,T),\, t \in (x, 2T - x)\}, \tag{1}$$

$$u_x(0,t) + ru(0,t) = 0, \qquad t \in [0, 2T], \tag{2}$$

$$u(x,x) = -1, \qquad x \in [0,T], \tag{3}$$

$$u(0,t) = g(t), \qquad t \in [0, 2T]. \tag{4}$$

The relations (1)-(3) are the direct problem, equality (4) is the additional information on the direct problem solution.

In correspondence of the inverse problem (1)-(3) we can state the following residual functional

$$J[q] = \int_0^{2T} [u(0,t) - g(t)]^2 dt \tag{5}$$

For the numerical solving of the direct problem we used the finite difference method.

The results of our numerical experiment will be presented.