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Numerical solving the Dirichlet problems for the Helmholtz equation using an analytical extension of the resolvent.

V.L. Lukinov

ICMMG SB RAS
Ak. Lavrent'ev prosp., 6,
630090 Novosibirsk, Russia
E-mail:Vitaliy.Lukinov@ngs.ru

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Let us consider the Dirichlet problem for the Helmholtz equation in a domain $D \subset R^3$ with boundary Γ:

$$(\Delta + c)u = -g, \quad u|_\Gamma = \varphi \tag{1}$$

Let us assume that the following conditions hold. The function g is satisfied Holder condition in $\overline{D}$, D is a bounded open set in R^3 with a regular boundary Γ, the function φ is continuous on Γ, $c < c^*$, where $-c^*$ is the first eigen value of Laplace operator defined on the domain D. These conditions provided the existence and uniqueness of a solution to the problem are supposed fulfilled and after change of all parametric functions by their modules.

It is known (see [1]), that under above stated conditions there is numerical Monte Carlo method named walks by spheres. Under $c < c^*$ the solution to the problem (1) is represented by convergent Neumann series. There are two aims of this paper. First, if parameter c is too close to c^* then the rate of convergence of the corresponding Neumann series and Monte Carlo method is too dissatisfaction. Second, if we have the following inequality $c > c^*$ then the Neumann series is diverged and we can't use the corresponding Monte Carlo method for the numerical solving problem (1).

We can use the analytical extension of the resolvent by the shift of the parameter c [3]. We hold the following equations

$$\begin{cases} (\Delta + (c - c_0))u_0 = -g, \quad u_0|_\Gamma = \varphi, \\ (\Delta + (c - c_0))u_i = -c_0 u_{i-1} - g, \quad u_i|_\Gamma = \varphi, \quad i \geq 1 \end{cases} \tag{2}$$

$$\begin{cases} (\Delta + (c - c_0))(u_1 - u_0) = -c_0 u_0, \quad u_1 - u_0|_\Gamma = 0, \\ (\Delta + (c - c_0))(u_i - u_{i-1}) = -c_0(u_{i-1} - u_{i-2}), \quad u_i - u_{i-1}|_\Gamma = 0, \quad i \geq 2 \end{cases} \tag{3}$$

$$\begin{cases} (\Delta + (c - c_0))^{i+1}(u_i - u_{i-1}) = (-1)^{i-1} c_0^i g, \\ (\Delta + (c - c_0))^i (u_i - u_{i-1})|_\Gamma = (-1)^i c_0^i \varphi, \\ (\Delta + (c - c_0))^k (u_i - u_{i-1})|_\Gamma = 0, \quad k = 0, \ldots, i-1. \end{cases} \tag{4}$$

If $c - c_0 < c^*$ and $\exists k > 0 : |c_0|^k \|(\Delta + (c - c_0)^{-k})\| = q < 1$ then the series $\sum\limits_{i=0}^{n} u_i - u_{i-1}$ tends to u when $n \to \infty$.

Using the parametric differentiation of the based on walks by spheres estimator of the solution to (1) with a special functions g, φ we constracted the corresponding estimator for the solution $u_i - u_{i-1}$ to (4)[2].

References

[1] Elepov B.S., Mikhailov G.A. About solution to the Dirihlet problem for equation $(\Delta + c)u = -g$ by constructed 'walk by spheres'. // J. Num. Math. and Math. Physics. – 1969. – No 3. – P. 647–654.

[2] Lukinov V.L., Mikhailov G.A., The probabilistic representation and Monte Carlo methods for the first boundary value problem for a polyharmonic equation. // Rus. J. Num. Anal. and Math. Modell. 2004, v. 19, N.5, p. 434-449

[3] Sabelfeld K.K. Monte Carlo Methods for boundary problems. – Moskow.: Nauka, 1989[Engl. transl.: Springer–Verlag, 1991].