International Conference "Inverse and Ill-Posed Problems of Mathematical Physics",
dedicated to Professor M.M. Lavrent'ev on occasion of his 75-th birthday, August 20-25, 2007, Novosibirsk, Russia

Numerical solution of spherical Radon problem with data on sphere

D.V. Nechaev*

* IG SB RAS,
Ak. Lavrentiev prosp., 15,
630090 Novosibirsk, Russia
E-mail: nechaev@hydro.nsc.ru

Spherical Radon problem [1] is a problem of recovering a function $q(\boldsymbol{r})$, $\boldsymbol{r} \in \mathbb{R}^3$ by its integrals over spheres

$$f(\boldsymbol{\rho}, t) = \frac{1}{4\pi t^2} \int_{|\boldsymbol{\rho}-\boldsymbol{r}|=t} q(\boldsymbol{r})\, d\boldsymbol{r}, \qquad \boldsymbol{\rho} \in \Gamma, \quad t > 0. \tag{1}$$

Problem (1) also known as the problem of recovering function by its spherical means [2].

Depending on type of the set Γ and assumptions about the function q various statements of the problem are possible. In this paper we assume that the function q is finite

$$\operatorname{supp} q \subset B = \{\boldsymbol{r} \in \mathbb{R}^3 : |\boldsymbol{r}| < 1\} \tag{2}$$

and bounded

$$q \in C(B); \tag{3}$$

and the set Γ coincides with the boundary of the ball B, i. e. is the unit sphere

$$\Gamma = \partial B = \{\boldsymbol{r} \in \mathbb{R}^3 : |\boldsymbol{r}| = 1\}. \tag{4}$$

This assumptions allow uniquely recover the function q using the data f, given for $t \in (0, 2)$. In the paper the explicit formula for the solution of the problem (1)–(4) is obtained (possible not for the first time):

$$q(\boldsymbol{r}) = -\frac{1}{2\pi} \Delta \int_{|\boldsymbol{\rho}|=1} |\boldsymbol{\rho} - \boldsymbol{r}| f(\boldsymbol{\rho}, |\boldsymbol{\rho} - \boldsymbol{r}|)\, d\boldsymbol{\rho}, \qquad \boldsymbol{r} \in B, \tag{5}$$

where Δ is the Laplace operator.

The algorithm of numerical solution of the problem based on formula (5) is developed and tested. The estimate of solution precision depending on data precision is given. To improve solution precision the procedure of smoothing the data is required. The influence of smoothing parameter on solution precision is investigated and expression for its optimal value is presented.

References

1. **T. Schuster, E. T. Quinto.** On a regularization scheme for linear operators in distribution spaces with an application to the spherical Radon transform. Preprint N 109. Universität des Saarlandes, Saarbrücken, 2004.

2. **Lavrentiev M. M., Saveliev L. Ya.** Operators theory and ill-posed problems. IM SB RAS, Novosibirsk, 1999.