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Quasi inversion of multishot - multioffset seismic data on the base of gaussian beams decomposition

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Abstract.

The paper is devoted to recovery of local rapid perturbations (scatterers/reflectors) of *a priori* known macrovelocity background by means of linearized asymptotic inversion of multi-shot multi-offset seismic data. Inversion procedure is done via application of some specific integral transform to input data with respect to source/receiver positions and time frequency. Result of this transform is represented as an asymptotic series with leading term being superposition of some specific spatial Fourier constituents of desired local perturbations. Composition of these constituents is totally determined by range of time frequencies and geometry of acquisition system. The approach is tested on Sigsbee2A synthetic data set. Numerical results are presented and discussed.

Introduction.

Inversion procedure presented below is destined for imaging of rapid variations of the earth velocities embedded within background with *a priori* known macrovelocity model. It should be noted that besides necessity to recover proper geometry of these structures it is very important to provide their ”true amplitude” imaging. As true amplitude imaging we mean images being free from influence of geometrical spreading produced by macro-velocity background. Currently the most popular approaches for this are based on Kirchhoff method (Kirchhoff migration, Kirchhoff-based inversion). Origin of these approaches can be traced to the paper *Beylkin (1985)*. Their principal limitations are introduced by the use of high-frequency asymptotic entailing assumption that response at point $\vec{r}$ to a point source at $\vec{r}\,'$ can be represented as

$$G(\vec{r},\vec{r}\,';\omega) \approx A(\vec{r},\vec{r}\,')exp\left[i\omega T(\vec{r},\vec{r}\,')\right] \tag{1}$$

with frequency-independent traveltime and amplitude. If this assumption is valid there is possible to integral transform (summation weights for discrete statement) providing one with a leading-order, high-frequency asymptotic inversion operator, that is with true amplitude quasi inversion procedure.

Representation (1) is inadequate for descriptions of some exploration seismic reflection data, but, besides, there are troubles connected with multivalued nature of travel time $T(\vec{r},\vec{r}\,')$ because of multipathing of the seismic energy. Retaining all the arrivals is difficult to achieve in a practical depth migration implementation, but even if these difficulties are overcome, there remains the problem with behavior of amplitude function within the area where the traveltimes are multivalued. These troubles could be overcome by Gaussian beam migration *(Hill, 2001)*, when Green’s function $G(\vec{r},\vec{r}\,';\omega)$ is computed by means of Gaussian beams superposition. Global regularity of GB provides a possibility to handle properly all singularities of ray fields and to get an uniform high frequency approximation of Green’s function. But at the moment there are only a few attempts to develop preserving amplitude version of Gaussian beam prestack migration.

Method

Statement

Let us suppose that a medium we are dealing with possesses *a priori* known macrovelocity constituent $c_0(x,z)$ and we are searching for its rapid perturbation $c_1(x,z)$.

Under some reasonable assumptions scattered/reflected wave field on the free surface (input multishot/multioffset data) can be represented as the following (Born's approximation):

$$\phi(x_r,x_s;\omega) = 2\omega^2 \int\int_X \frac{1}{c_0^2(\xi,\eta)} \cdot \frac{c_1(\xi,\eta)}{c_0(\xi,\eta)} G_0(\xi,\eta;x_s,0;\omega) G_0(x_r,0;\xi,\eta;\omega) d\xi\eta. \tag{2}$$

Here $(x_r,0)$, $(x_s,0)$ are receiver and source positions respectively and $G_0(\xi,\eta;x,z;\omega)$ - is Green's function for macro-velocity model.

Input data:

$$\phi(x_r,x_s,\omega) = u_{sc}(x_r,0;x_s,0;\omega) : 0 < \omega_1 \le \omega \le \omega_2; \quad X_{0s} \le x_s \le X_{1s}; \quad X_{0r} \le x_r \le X_{1r}. \tag{3}$$

The problem is to resolve linear integral equation (2) with respect to function $\frac{c_1}{c_0}$ for given macro-velocity model c_0 and seismic data (3).

Asymptotic inversion

Let us fix some interior point $\overline{x} = (x_i, z_i)$ and shoot a couple of Gaussian beams - later referred as left and right (Fig. 1) - towards acquisition system. Twice application of Green's and reciprocity theorems leads to the following integral equation:

$$2\omega^2 \int\limits_X K(\overline{x};\overline{y};\omega) \frac{c_1(\overline{y})}{c_0(\overline{y})} dy == \int\limits_{x_s}\int\limits_{x_r} \tau_s^{(gb)}(x_s;\omega)\tau_r^{(gb)}(x_r;\omega)\phi(x_r;x_s;\omega)dx_s dx_r. \tag{4}$$

The kernel of integral operator in the left-hand side of this equation is product of a couple of mentioned above Gaussian beams:

$$K(y,\overline{x};\alpha,\beta;\omega) = u_s^{(gb)}(y;\overline{x};\alpha,\beta;\omega) u_r^{(gb)}(y;\overline{x};\alpha,\beta;\omega);$$

while integral transform of input multishot/multioffset data is defined by the following functions:

$$\tau_{s(r)}^{(gb)}(x_{s(r)};\omega) = \left.\frac{\partial u_{s(r)}^{(gb)}(x_{s(r)},z;\overline{x};x_{0s(0r)};\omega)}{\partial z}\right|_{z=0}.$$

Let us now apply Fourier transform with respect to time frequency ω to both sides of integral equation (4) and compute it for time $t(\overline{x};\alpha,\beta) = \tau_s(\overline{x};\alpha,\beta) + \tau_r(\overline{x};\alpha,\beta)$. Here $\tau_s(\overline{x};\alpha,\beta)$ and $\tau_r(\overline{x};\alpha,\beta)$ are travel-times from $\overline{x}$ to free surface computed for a priori known macrovelocity model along left and right rays respectively. For the next step we apply integration with respect to dip angle α under fixed opening angle β. These transformations lead to the linear integral operator M^β in the left-hand side which can be represented as the series of pseudodifferential ones:

$$M^\beta = T_0^\beta + T_1^\beta + T_2^\beta + T_3^\beta + \ldots$$

It should be noted that operator T_n^β acts from C^k to C^{k+n}. As we are interested in sharp perturbations of macrovelocity only, we can neglect all terms of this series except of the very first one and consider left-hand side as application of the following simple transform to desired perturbation:

$$M^\beta < \frac{c_1}{c_0} > (\overline{x}) \approx T_0^\beta < \frac{c_1}{c_0} > (\overline{x}) = \int\int\limits_{X_{par}(\overline{x})} d\overline{p} \cdot \int\int\limits_X \exp\{i \cdot \overline{p} \cdot \overline{x}\} \exp\{-i \cdot \overline{p} \cdot \overline{y}\} \cdot \frac{c_1(\overline{y})}{c_0(\overline{y})} d\overline{y}. \tag{5}$$

The first integration in the right hand side of (5) is performed over domain X_{par} :

$$X_{par}(x_i, z_i) = \{\overline{p} : \omega_1 \leq \frac{|\overline{p}|c_0(\overline{x})}{2\cos(\beta)} \leq \omega_2; \alpha_1 \leq \ \arctan -\frac{p_x}{p_z} \leq \alpha_2\}. \tag{6}$$

One can easy see that operator T_0^β is superposition of forward and quasi-inverse two-dimensional Fourier transform of the function $\frac{c_1}{c_0}$. We call it *quasiinverse* because it is performed not over the whole spectral space, but over it subspace X_{par} only. That is, we will image properly only constituents of reflectors/scatterers which possess spatial spectrum within specific set of partial reconstruction (6).

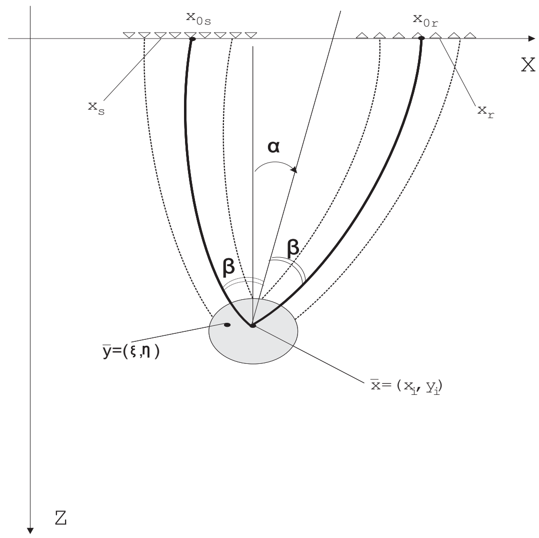

Figure 1: Geometry of true amplitude Gaussian beam imaging.

Numerical examples

Presented above procedure of true amplitude imaging was used on synthetic data set Sigsbee2a calculated by SMAART Joint Venture. The total stratigraphic model is presented on the Fig.2. True amplitude images are presented on the Fig.3 for target area out of salt body and on the Fig.4 for target area under salt intrusion. We would like to pay attention on the quite well recovery of faults for both target areas. One can estimate that intensity of images are proportional to the real distribution of reflectivity as well(real model possesses intensity of reflectors proportional to fast velocity perturbation).

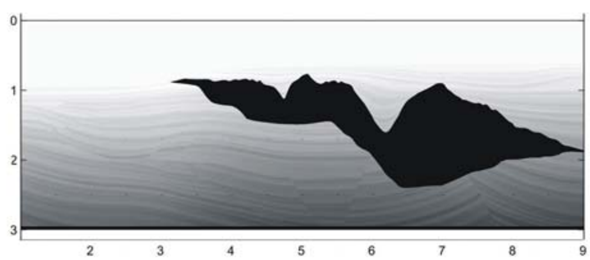

Figure 2: Total stratigraphic model Sigsbee2A model. Horizontal and vertical axes - distance in feets$\times 10^4$

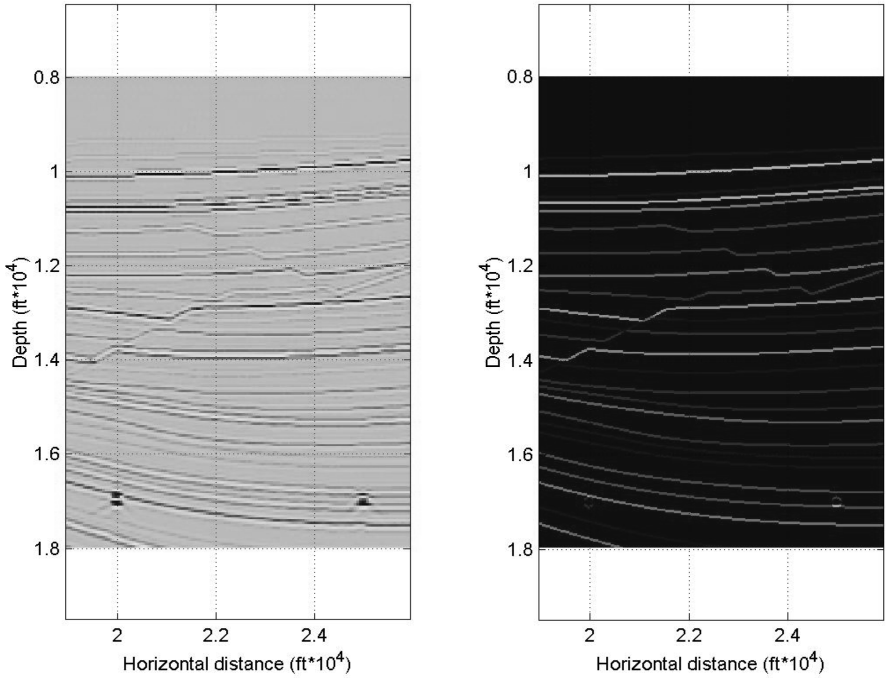

Figure 3: Recovered (left) and real (right) images of target area out of salt intrusion.

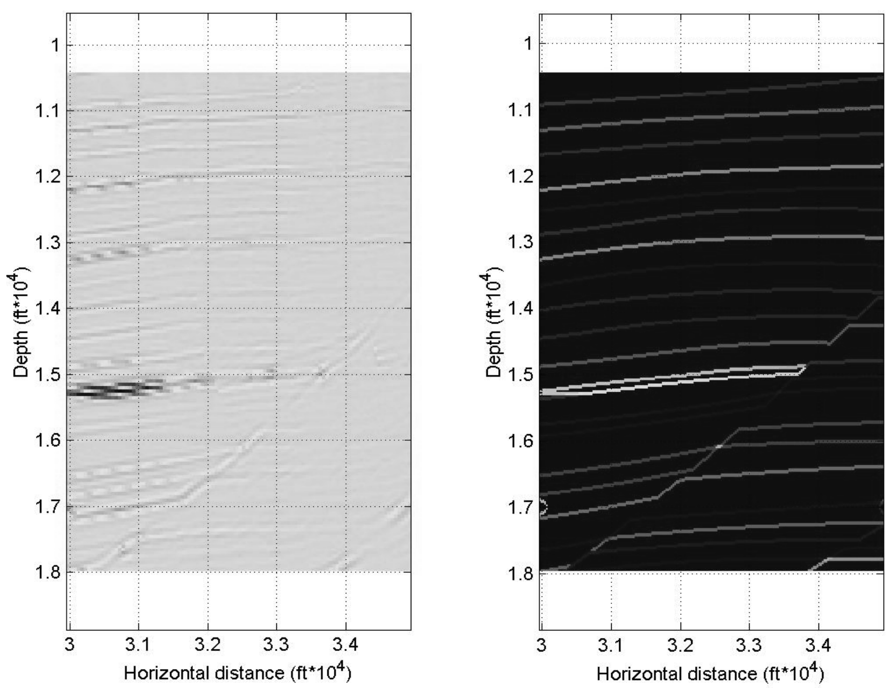

Figure 4: Recovered (left) and real (right) images of subsalt target area.

Conclusion

It should be noted that proposed imaging procedure provides so called ”selective image”. Their main features are as follows:

- If spatial spectrum of reflector/scatterer component lies within a set of partial reconstruction this perturbation is totally recovered ;
- If spatial spectrum of a local object is out of a set of partial reconstruction, it happens to be totally ”invisible”;
- If spatial spectrum of a local object possesses nonempty intersection with a set of partial reconstruction selective image will be made from linear projection of desired perturbation onto the image of a set of partial reconstruction.

This follows very useful property - any singular object like diffractor/scatterer, crack, fault, pinch and so on possesses rather wide spatial spectrum and, so, will be presented for a range of selective images. On the contrast, any regular interface possesses rather narrow spatial spectrum and, so, one can easy choose dip and open angles providing the set of partial reconstruction being far away from this spatial spectrum. This opens a possibility to get reliable image of low contrast singular objects of subseismic scale.

References

[1] Beylkin, G., 1985, Imaging of discontinuous in the inverse scattering problem by inversion of causual generalized Radon transform, *J. Math. Phys.*, **v.26(1)**, pp.99 - 108.

[2] Bleistein N., 1987, On the imaging of reflectors in the Earth, *Geophysics*, **52**, 931 - 942.

[3] Cerveny, V., & Popov, M.M., & Psencik, I., 1982, Computation of wave fields in inhomogeneous media. Gaussian beam approach *Geoph. J. R. Astr. Soc.*, **v.70**, pp. 109-128.

[4] Shubin M.A., 1978, Pseudodifferential operators and spectral theory, Moscow,”Nauka”.(in Russian)