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Comparative analysis of methods for inverse acoustic problems solution

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Let us consider the following problem

$$c^{-2}(z)v_{tt} = v_{zz} - \frac{\rho'(z)}{\rho(z)}\, v_z, \qquad z > 0, \quad t > 0; \tag{1}$$

$$v\big|_{t<0} = 0, \qquad v_z\big|_{z=+0} = \delta(t), \tag{2}$$

$$v(+0,t) = f(t). \tag{3}$$

Here $c(z) \geq c_0 > 0$ ($c_0 = \text{const}$) is the velocity of wave propagation; $\rho(z) \geq \rho_0 > 0$ ($\rho_0 = \text{const}$) is the density; $v(z,t)$ is the acoustic pressure; $\delta(t)$ is Dirac delta-function.

Forward (*direct*) problem (1), (2) is to find solution of equation (1) $v(z,t)$ by known functions $c(z)$ and $\rho(z)$.

Inverse problem (1)–(3) is to find functions $v(z,t)$, $c(z)$ and $\rho(z)$ by known $f(t)$.

Let us introduce "travel-time" variable

$$x = \psi(z), \quad \psi(z) = \int_0^z \frac{\mathrm{d}\xi}{c(\xi)}.$$

The function $\psi^{-1}(x) = z$ exists such as $c(z) > 0$ and therefore we can introduce new functions $u(x,t) = v(\psi^{-1}(x),t)$, $\sigma(x) = c(\psi^{-1}(x))\rho(\psi^{-1}(x))$.

We reduce (1)–(3) to the following inverse problem:

$$u_{tt} = u_{xx} - \frac{\sigma'(x)}{\sigma(x)}\, u_x, \qquad x > 0, \quad t > 0; \tag{4}$$

$$u\big|_{t<0} = 0, \quad u_x\big|_{x=+0} = c(+0)\delta(t); \tag{5}$$

$$u(+0,t) = f(t), \tag{6}$$

where $v(x,t) = u(z,t)$ is the exceeded pressure; $\sigma(x) > 0$ is the acoustic impedance; $c(+0)$ is known. In order to solve (4)–(6) we have to find functions $u(x,t)$ and $\sigma(x)$ by known additional information (6) about the forward problem solution (4), (5).

We consider the following statements of inverse problem (4)–(6): operator, differential, finite-difference, variational and integral.

For recovering unknown coefficient $\sigma(x)$ in inverse problem (4)–(6) we apply the following methods

- Finite-difference scheme inversion;
- Gel'fand-Levitan-Krein method;
- Boundary Control method;

- Gradient methods (the steepest descent, Landweber iterations);
- Newton–Kantorovich method.

The theoretical and numerical results will be presented and discussed.

References

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