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The Inverse Problem of Reconstruction of the Original Tsunami Waveform

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The inversion problem to determine the initial sea perturbation is considered as the usually ill-posed problem of the hydrodynamic inversion of tsunami tide-gage records. The wave propagation is described by the linear shallow-water equations when the depth is assumed as an arbitrary function of two variables. The direct problem is approximated by the explicit-implicit finite difference scheme. The ill-posed inverse restoration problem is regularized by means of the least square inversion using the truncated SVD approach.

Mathematically, the initial tsunami waveform problem is formulated as inverse problem of mathematical physics for restoration of the initial water displacement in the source area by the water level oscillations observed on a number of points distributed in the ocean. The possibility to get an unique solution ([1]) exists only when the function of the source allows the factorization, i. e., the dependencies on time and space variables are separated. Furthermore, the dependence on time is a priori given. We assume that the time dependency is described by the Heavyside function. It's well known that the above formulated problem is an ill-posed problem. The mathematical description of the direct problem of the wave propagation consists in the linear shallow-water system of differential equations in the rectangular coordinates. This system is approximated by the explicit-implicit finite difference scheme on a uniform rectangular grid, so the system of the linear algebraic equations is obtained. The ill-posed inverse problem of the reconstruction is regularized by means of the least square inversion, using truncated SVD approach - in this method the inverse operator is regularized with the help of its restriction on the subspace spanned on a finite sample of the first right singular vectors (see [4]). So-called r-solution (see [2]) is a result of the numerical process. The quality of the solution obtained is evaluated as relative errors (in L_2-norm) in restoration of the source function. The results are fairly satisfactory, if the receivers have a good azimuthal coverage with respect to source area. This dependence is investigated in the present paper by way of the numerical simulation. The quality of the solution obtained is evaluated as relative errors (in L_2-norm) in restoration of the source function. We neglect the curvature of the Earth and we assume that z-axis be directed downwards with depth. Since the tsunami wave in the ocean is a long gravitational low-amplitude wave, its propagation can be described by the shallow water equation:

$$W_{tt} = \mathrm{div}\,(\mathrm{h(x,y)grad\,W}) + \mathrm{f_{tt}(t,x,y)}; \tag{1}$$

with the initial conditions

$$W_{|t=0} = 0; \quad W_{t|t=0} = 0; \tag{2}$$

and the boundary conditions

$$W_{|\Gamma} = W_0(x(s), y(s), t), \tag{3}$$

where $W(x,y,t)$ is a water elevation over the undisturbed state, $h(x,y)$ is the depth of the ocean, $f(x,y,t)$ describes the movement of the bottom in the tsunami area. Here we consider run ups as normal vectors

arrivals (on coast line L). The velocity of the tsunami wave propagation is also described as $c(x,y) = \sqrt{gh}$. We solve the problem in the domain with piecewise-linear inner and outer boundaries. An unique solution exists only when the function of the source allows factorization, i.e., the function $f(x,y,t)$ can be factorized as $f(x,y,t) = \varepsilon(t) \cdot \varphi(x,y)$, where $\varepsilon(t)$ is the Heavyside function, $\varphi(x,y)$ is the bottom movement in the tsunami center (subdomain Ω). Let us assume that the support of the function $\phi(x,y)$ is included in the rectangle Φ and this function is of the class ${W_2}^1(\Phi)$. So, that the solution of the problem (1)-(3) is now reduced to the following vector equation:

$$\mathcal{A} < \phi(x,y) >= U(t) \tag{4}$$

We can consider $\mathcal{A} : {W_2}^1(\Phi) \to L_2(M \times (0,T))$. As it was done in [3], it is possible to prove that this operator is a compact one, so it does not possess a bounded inverse. The numerical solution to equation (4) includes its regularization using the SVD-decomposition of the operator $\mathcal{A}$, that leads to the construction of r-solution (see [3]). The unknown function $\varphi(x,y)$ will be sought in the form of a series of spatial harmonics with the unknown coefficients in Ω, when the given data is the water elevation $W_0(x_i,y_i,t)$ in a certain set of the receivers $\{M : (x_i,y_i), i = \overline{1,P}\}$, disposed on the line Γ. The algorithm used in this way and substantiation of using this approach are described in detail in [4]. The system of equations (1)-(3) is approximated by the explicit-implicit finite difference scheme on the uniform rectangular grid based on the 4-point pattern. The scheme is of second order of accuracy with respect to spatial variables and of the first order with the respect to time. are described in detail in [4].

Numerical experiments are presented for the model bottom relief having some basic morphological features typical for the island arc regions, with inner and outer boundaries of the target domain. As a model of initial water displacement we used displacement representing the bottom deformation due to the typical tsunamigenic earthquakes with reverse dip-slip or low-angle trust mechanisms. A series of calculations was carried out by the method proposed and were aimed at recovering the unknown function $\varphi(x,y)$. The observed data concerning the form of the arrived wave were simulated as a result of solution to direct problem (1)-(3), perturbed by the background noise, i.e., a high-frequency disturbance rate of 5% of a maximum amplitude of a signal over all the receivers. It is necessary to recognize that the results obtained strongly depend on the presence of disturbance due to the ill-posedness of the problem. However, since a tsunami wave is considered to be of essentially lower frequency as compared to the background noise, it is reasonable enough to apply the frequency filtration of the observed signal with algorithm as in [5]. We have shown that to attain a reasonable quality of the source restoration in this case we need, at least seven records distributed over the space domain and their azimuthal coverage plays the key role in obtaining the satisfactory results of inversion.

Список литературы

[1] Kaistrenko V. M. :'Inverse problem for reconstruction of tsunami source'. In: In: *Tsunami waves. Proc. Sakhalin Compl.Inst.* 1972, is.29. P.82-92.

[2] Cheverda V. A., Kostin V.I.: 'r-pseudoinverse for compact operators in Hilbert space: existence and stability'. In: *J. Inverse and Ill-Posed Problems*, 1995, V.3, N.2, pp. 131-148.

[3] T.A.Voronina and V.A.Tcheverda :'Reconstruction of tsunami initial form via level oscillation'. In: *Bull.Nov.Comp.Center,Math.Model.in Geoph.*, 4(1998), p.127-136

[4] T.A.Voronina :' Reconstruction of initial tsunami waveform by the coastal observations invertion'. In: *Bull.NCC, Math.Model.in Geoph.*, 7(2002), p.89-100.

[5] Belonosov A.S., Tsetsokho V.A. The numerical algorithm and procedure of smoothing functions, approximatte given at grid noids of a irregular meash on the plane. -in book: Some problems of math. physics and interpretation of the geophysical observations. CC SOAS USR, 1976,p.6-29