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Contact problems for bodies of different dimensions

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The talk is concerned with analysis of unilateral contact problems between two inclined elastic plates and between a plate and a beam. Considered problems are characterized by a contact set having a dimension less than one of that of a domain. This property leads to a new class of free boundary problems with inequality type boundary conditions. The main attention is paid to a suitable description of boundary conditions along the contact zone. Asymptotic properties of solutions are established provided that elasticity parameters of the contacting bodies are changing, in particular, in the case when a stiffness of the elastic body goes to infinity.

Let $\Omega \subset R^2$ be a bounded domain with smooth boundary Γ. We assume that Ω corresponds to the middle surface of an elastic plate. A unit external normal vector to Γ is denoted by $q = (q_1, q_2)$. Another elastic plate with a middle surface D is situated at angle α with respect to the first one, $\alpha \in (0, \frac{\pi}{2}]$. The domain D is assumed to be bounded, and its boundary is denoted by ∂D, $\Omega \cap D = \emptyset$, $\Omega \cap \partial D \neq \emptyset$. Denote $\gamma_0 = (\partial D) \setminus \Omega$. In this case $\partial D = \gamma \cup \bar{\gamma}_0$.

Let $\nu = (\nu_1, \nu_2)$ be a unit normal vector to γ located in the plane Ω. By $n = (n_1, n_2)$ we denote a unit normal external vector to ∂D located in the plane D. Also assume that ∂D is a smooth curve, γ is a connected set, and γ does not intersect the boundary Γ of the domain Ω.

Differential formulation of the contact problem between two inclined plates is as follows. It is necessary to find functions $v(x)$, $w(y)$, $x = (x_1, x_2) \in D, y = (y_1, y_2) \in \Omega_\gamma$, such that

$$\Delta^2 v = g \ \text{ in } \ D, \tag{1}$$

$$\Delta^2 w = f \ \text{ in } \ \Omega_\gamma, \tag{2}$$

$$v = v_n = 0 \ \text{ on } \ \gamma_0, \tag{3}$$

$$w = w_q = 0 \ \text{ on } \ \Gamma, \tag{4}$$

$$v \cos\alpha - w \geq 0, \ [t^\nu(w)](v\cos\alpha - w) = 0 \ \text{ on } \ \gamma, \tag{5}$$

$$[w] = [w_\nu] = 0, \ [m(w)] = 0 \ \text{ on } \ \gamma, \tag{6}$$

$$[t^\nu(w)] \leq 0, \ m(v) = 0, \ [t^\nu(w)]\cos\alpha = t^n(v) \text{ on } \ \gamma. \tag{7}$$

Here $[u] = u^+ - u^-$ is a jump of a function u on γ, where $u^\pm$ correspond to the positive and negative (with respect to ν) faces $\gamma^\pm$, respectively,

$$v_n = \frac{\partial v}{\partial n}, \ w_q = \frac{\partial w}{\partial q}, \ w_\nu = \frac{\partial w}{\partial \nu},$$

$$m(w) = \kappa_1 \Delta w + (1-\kappa_1)\frac{\partial^2 w}{\partial \nu^2}, \ \ t^\nu(w) = \frac{\partial}{\partial \nu}\left(\Delta w + (1-\kappa_1)\frac{\partial^2 w}{\partial s^2}\right),$$

where κ_1 is the Poisson ratio of the horizontal plate, $(s_1, s_2) = (-\nu_2, \nu_1)$. The values $m(v), t^n(v)$ are introduced similar to those of $m(w), t^\nu(w)$,

$$m(v) = \kappa_2 \Delta v + (1 - \kappa_2)\frac{\partial^2 w}{\partial n^2}, \quad t^n(v) = \frac{\partial}{\partial n}\left(\Delta v + (1 - \kappa_2)\frac{\partial^2 v}{\partial \tau^2}\right)$$

with the Poisson ratio κ_2 for the inclined plate, $(\tau_1, \tau_2) = (-n_2, n_1)$.

The functions $g \in L^2(D)$, $f \in L^2(\Omega)$ are given.

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