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Inverse and optimization problems for cracks

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In the framework of an optimization approach to brittle fracture, we consider evolution in time-parameter t of crack Γ_C in domain $\Omega \subset \mathbb{R}^N$, $N = 2, 3$, as a solution of the global minimization problem:

Find $\Gamma_C(t) \in \Sigma(\Omega)$ for $t \geq 0$, such that

$$u(t) \in K(\Omega \setminus \Gamma_C(t)), \quad T(u(t); \Omega \setminus \Gamma_C(t)) \leq T(v; \Omega \setminus \Gamma_C(t)) \quad \text{for all } v \in K(\Omega \setminus \Gamma_C(t));$$

$$\Gamma_C(t) \supset \bigcup_{s<t} \Gamma_C(s), \quad T(u(t); \Omega \setminus \Gamma_C(t)) \leq T(u; \Omega \setminus \Gamma_C) \quad \text{for all } \Gamma_C \supset \bigcup_{s<t} \Gamma_C(s),$$

$$u \in K(\Omega \setminus \Gamma_C), \quad T(u; \Omega \setminus \Gamma_C) \leq T(v; \Omega \setminus \Gamma_C) \quad \text{for all } v \in K(\Omega \setminus \Gamma_C).$$

Here $u \mapsto T(u; \Omega \setminus \Gamma_C) : K(\Omega \setminus \Gamma_C) \mapsto \mathbb{R}$ is a cost functional of the total potential energy of the solid with crack, occupying the domain $\Omega \setminus \Gamma_C$, over the set of admissible displacements $u \in K(\Omega \setminus \Gamma_C)$, and $\Sigma(\Omega)$ is the set of admissible crack paths in Ω.

The problem deals with finding optimal shape parameters presented by geometrical or topological variables of a domain with unknown crack. For the pre-defined crack path along the curve $\Sigma \in \Sigma(\Omega)$ in $\mathbb{R}^2$, respective one-parametric optimization problem is well posed with respect to the length parameter of the crack $\Gamma_C(t)$. It describes appearance and quasi-static propagation of the crack, for instance, during the delamination process [4], or by quasi-brittle fracture [7]. A two-parametric optimization problem stated with respect to unknown shape parameters of the crack length and the angle of its kink is studied in [2].

The proper modeling of equilibrium of solids with cracks is given in [1] in framework of variational formulation, accounting conditions of non-penetration between the crack surfaces in the admissible set $K(\Omega \setminus \Gamma_C)$, which results in constrained crack problems: Find $u \in K(\Omega \setminus \Gamma_C)$, such that

$$T(u; \Omega \setminus \Gamma_C) \leq T(v; \Omega \setminus \Gamma_C) \text{ for all } v \in K(\Omega \setminus \Gamma_C).$$

The necessary ingredients of the optimization formulation include kinematic description of cracks as codimensional-1 open manifolds. The crack evolutions with given a-priori vector of the velocity field $V \in C([0, L]; W^{1,\infty}(\Omega))^N$ can be managed in two ways: constructing homeomorphic maps between domains with cracks and solving nonlinear ODE, or by implicit surface functions which solve a linear transport equation with V.

To find the global minimum, optimization algorithm is realized in a constructive way on the set of extremal points of T by the path-following method, which uses the differentiability properties of the reduced cost functional $\Gamma_C(t) \mapsto T(u(t); \Omega \setminus \Gamma_C(t))$. This approach requires sensitivities of the crack problems with respect to perturbations of parameters of the crack shape, in particular, to find the shape derivative (the energy release rate at the crack tip) by means of the directional derivative $-T'_V$ [3].

Based on the generalized differentiability property, a semi-smooth Newton method proposed in the form of primal-dual active-set strategy (PDAS-method) is used in [5] as the efficient numerical technique for solution of the constrained minimization problems, in particular with cracks, due to the property of its

unconditional global and, moreover, monotone convergence. Solving numerically the optimization problem we present examples of quasi-static propagation of cracks [6], which are based on the refinement of the Griffith fracture law in the frame of the optimization approach.

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