International Conference "Inverse and Ill-Posed Problems of Mathematical Physics",
dedicated to Professor M.M. Lavrent'ev on occasion of his 75-th birthday, August 20-25, 2007, Novosibirsk, Russia

Pressure Transient Analysis in Heterogeneous Oil Reservoirs

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Introduction

Pressure transient analysis is one of the main tools used by reservoir engineers to manage the development of a field. From these data, it is possible to determine the production potential and geologic characteristics of different rocks, evaluate hydrocarbon reserves and forecast future performance. In order to develop techniques for interpretation of pressure data and obtain reservoir characteristics, it is very important the knowledge of basic concepts of fluid flow through porous media and inverse problems. In this presentation, it will be shown some theory of fluid flow in porous media in heterogeneous reservoir and its application in ressure transient analysis.

Physical Problem

The permeability distribution can be calculated by methods which will estimate permeability from well-test pressure data. Oliver [2] used a pertubation theory techique to obtain the wellbore pressure drawdown solution at a single well in a infinite-acting reservoir where absolute permeability varies with position. His solution assumes 2D flow in a (r, θ) coordinate system and the permeability is a function of r and θ and varies slightly about a average value, k_{ref}. The Backus-Gilbert method was used to aproximate the permeability distribution under the assumption that a reference permeability value can be determinated from a semi-log of pressure vs. time. Another method found in literature also calculates the permeability distribution directly from well-test pressure data [1]. The Inverse Solution Algorithm (ISA) shows that the base permeability value controls only the shifting of the time scale used to evaluate the kernel weigthing function in Olivers solution. The algorithm can be applied for large variations in permeabilities and in cases where pressure data exhibit no semilog straight lines. Rosa and Horne [3] examined the same problem as Oliver. They concluded that the inverse problem (the determination of permeability distributions) does not have a unique solution. For a multicomposite reservoir, the permeability distribution could be determined only if the inner and outer radii of each zone are known.

Yeh and Agarwal [4] considered the analysis of pressure-falloff data obtained after water injection into an oil reservoir under radial flow conditions. They assumed that the value of pressure derivative represents the volumetric average from the wellbore to the radius of investigation. The Yeh-Agarwal procedure can be modified and applied to obtain the permeability distibution under single-phase flow conditions. The basic model assumes that the permeability distribution depends only on distance from the well and porosity is constant. However, if the porosity variation is small compared with the permeability variation, which is normally the case, assuming a constant porosity value will have a only a small effect on the accuracy of the inverse problem.

It is comsidered single-phase flow of a slightly compressible fluid of a constant compressibility and viscosity to a well in the center of a cylindrical reservoir of uniform thickness. The production rate is constant and the outer boundary is infinite. Gravity and wellbore storage effects are neglected and the vertical pressure gradients are negligible. If we assume that rock properties depend only on distance from the well, then flow is radial.

The mathematical model, in dimensionless variables, can be written as

$$\frac{1}{r_D}\frac{\partial}{\partial r_D}\left[r_D k_D(r_D)\frac{\partial p_D}{\partial r_D}\right] = \frac{\partial p_D}{\partial t_D} \tag{1}$$

Initial condition:

$$p_D(r_D, 0) = 0 \tag{2}$$

Inner boundary condition:

$$\left[r_D k_D(r_D)\frac{\partial p_D}{\partial r_D}\right]_{r_D=1} = -1 \tag{3}$$

Outer boundary conditions:

$$\lim_{r_D\to\infty} p_D(r_D, t_D) = 0 \tag{4}$$

The first aproximate solution to this problem was presented by Oliver. Defining a pertubation to the permeability parameter as

$$k_D(r_D) = \frac{1}{[1-\varepsilon f(r_D)]} \tag{5}$$

and developing $k_D(r_D)$ in Taylor series around the unit value

$$k_D = 1 + \varepsilon f + \varepsilon f + ... \tag{6}$$

and then pertubation technique together with Laplace transform is used. Developing the resulting equation to the wellbore both the wellbore pressure and its derivative solution can be written in real space, respectively, as:

$$p_{wD}(1, t_D) = \frac{1}{2}\ln\left(\frac{4t_D}{e^{\gamma}}\right) - \int_1^{\infty} G(\xi, t_D)\left[1 - \frac{1}{k_D(\xi)}\right]d\xi \tag{7}$$

and

$$p'_{wD}(1, t_D) = \frac{\partial p_D(1, t_D)}{\partial \ln t_D} = t_D\frac{\partial p_D}{\partial t_D} = \frac{1}{2} - \int_1^{\infty} K(\xi, t_D)\left[1 - \frac{1}{k_D(\xi)}\right]d\xi \tag{8}$$

Generalized Pressure Derivative Solution (GPDS)

Feitosa [1] developed a pressure derivative solution that is valid for any arbitrary variation in permeability, eiher small or large. The derivation proceeds directly from Oliver's approximate solution. For the sake of simplicity, we define $k(r)$ to be the harmonic average of $k(r, \theta)$.

$$\frac{1}{k(r)} = \frac{1}{2\pi}\int_0^{2\pi}\frac{1}{k(r,\theta)}d\theta \tag{9}$$

With this notation, Oliver's solution can be written as

$$p'_{wD}(1,t_D) = \frac{\partial p_D(1,t_D)}{\partial \ln t_D} = t_D \frac{\partial p_D}{\partial t_D} = \frac{1}{2} - \int_1^\infty K(r_D,t_D)\left[1 - \frac{1}{k_D(r_D)}\right] dr_D \tag{10}$$

The matematical definition of the kernel function $K(r_D,t_D)$ is given by:

$$K(r_D,t_D) = \frac{\sqrt{\pi}}{2}\frac{r_D}{t_D}\exp\left(\frac{-r_D^2}{2t_D}\right) W_{\frac{1}{2},\frac{1}{2}}\left(\frac{r_D^2}{t_D}\right) \tag{11}$$

If $k(r) = k_{ref}$ for all r, then the integral on the right side of equation 10 vanishes and it reduces to the standard homogeneous reservoir result. If, on the other hand, permeability is constant with $k(r) = k_1$, for all r, but with base dimensionless pressure on k_{ref}, then equation 10 results

$$p'_{wD}(t_D) = \frac{1}{2}\frac{k_{ref}}{k_1} \tag{12}$$

It is easy to see that equation 10 reduces to equation 12 if and only if

$$\int_1^\infty K(r_D,t_D) dr_D = \frac{1}{2} \tag{13}$$

Using equation 13, we obtain from equation 10

$$p'_{wD}(t_D) = \int_1^\infty K(r_D,t_D)\left(\frac{1}{k_D(r_D)}\right) dr_D \tag{14}$$

Using the definition of instantaneous permeability

$$\hat{k} = \frac{1}{2}\left(\frac{70.6 q\mu}{h\Delta p'_w}\right) \tag{15}$$

equation 14 becomes

$$\frac{1}{2\hat{k}} = \frac{h\Delta p'_w}{141.2 q\mu}\int_1^\infty K(\xi,t_D)\left[\frac{1}{k_D(\xi)}\right] d\xi \tag{16}$$

The most important observation is that the equation 16 can be more accurate if we replace the dimensionless time, t_D, which is based on k_{ref}, by the dimensionless pseudotime, which is based on $\hat{k}$.

$$\hat{t}_D = \frac{\hat{k}t}{\phi\mu c_t r_w^2} \tag{17}$$

Thus, equation 16 is modified to

$$\frac{1}{\hat{k}} = \frac{h\Delta p'_w}{70.6 q\mu}\int_1^\infty K(r_D,\hat{t}_D)\left[\frac{1}{k_D(r_D)}\right] dr_D \tag{18}$$

Hence, equation 18 is independent of any reference permeability.

Inverse Solution Algorithm

The inverse solution algorithm is recursive. Effectively, we determine the permeability at a sequence of radial locations. To start the algorithm, equation 15 is applied to compute $\hat{k}(t_1)$ where t_1 is the first time point at which there is accurate pressure derivative data. With these permeability and time values, we then compute correspondin value of $\hat{t}_D$ from equation 17. Next r_1 is calculated by setting $i = 1$ in the following radius of investigation equation:

$$r_i = 2r_w\sqrt{\hat{t}_D} \tag{19}$$

Assuming that $k(r) = k(r_1)$ for $rw \leq r \leq r_1$, we define $r_0 = r_w$.

The described steps are repeated until the values of $\hat{k}$, $\hat{t}_D$, r corresponding to all n values of pressure derivative points are calculated. With these concepts, the ISA sequentially calculates the permeability distribution for each zone from the following relation:

$$\Delta\left(\frac{1}{k_n}\right) = \frac{\frac{1}{\hat{k}_n} - \frac{1}{k_{n-1}} + \sum_{i=2}^{n-1}\left[\Delta\left(\frac{1}{k_i}\right)\int_{\hat{z}_{0D}}^{\hat{z}_{i-1D}} \Omega(\hat{z}_D) d\hat{z}_D\right]}{1 - \int_{\hat{z}_{0D}}^{\hat{z}_{n-1D}} \Omega(\hat{z}_D) d\hat{z}_D} \tag{20}$$

where

$$\hat{z}_D = \frac{r_D}{\sqrt{\hat{t}_D}} \tag{21}$$

$$\hat{z}_{0D} = \min\left\{\frac{1}{\sqrt{\hat{t}_D}}, 0.12\right\} \tag{22}$$

and

$$\Omega(\hat{z}_D) = \hat{z}_D \sqrt{\pi} \exp\left(-\frac{\hat{z}_D^2}{2}\right) W_{\frac{1}{2},\frac{1}{2}}\left(\hat{z}_D^2\right) \tag{23}$$

Yeh-Agarwal Procedure

In their work, Yeh-Agarwal [4] assumed that the total mobility computed from the instantaneous value of the pressure derivative represents the volumentric average from the wellbore to the radius of investigation. Here, we simply present the modified procedure for the analysis of drawdown data.

Step 1.Compute $\hat{k}$ as a function of time from equation 15,

Step 2.Compute corresponding radii of investigation r_i by

$$r_i = 1.5\sqrt{\frac{\hat{k}t}{\phi \mu c_t}} \tag{24}$$

Step 3.Compute the reservoir permeability at r_i from

$$k(r_i) = \frac{r_i}{2}\frac{d\hat{k}(r_i)}{dr_i} + \hat{k}(r_i) \tag{25}$$

As in the original Yeh-Agarwal method for analysis of pressure falloff data, the preceeding procedure assumes that $\hat{k}$ calculated from equation 15 represents a volumetric average of the actual permeability distribution whithin the region between the wellbore and the radius of investigation:

$$\hat{k} = \frac{2}{r_i^2}\int_{r_w}^{r_i} k(r) r dr \tag{26}$$

On the basis of Olivers result, Feitosa assumed that the algorithm can be improved by assuming that $\hat{k}$ is based on a harmonic average instead of a volumetric average:

$$\frac{1}{\hat{k}} = \frac{2}{r_i^2}\int_{r_w}^{r_i} \frac{1}{k(r)} r dr \tag{27}$$

Under this assumption, equation 25 should be replaced by

$$\frac{1}{k(r_i)} = \frac{1}{\hat{k}(r_i)} - \frac{r_i}{2}\frac{1}{\hat{k}^2(r_i)}\left[\frac{d\hat{k}(r)}{dr}\right]_{r_i} \tag{28}$$

Table 1: Reservoir Parameters

$k_1 = 20\,mD$	$r_1 = 32\,ft$	$h = 20\,ft$	$\mu = 1.0\,cP$
$k_2 = 30\,mD$	$r_2 = 151\,ft$	$\phi = 0.20$	$c_t = 10 \times 10^{-6}\,psi$
$k_3 = 10\,mD$	$r_3 = 534\,ft$	$p_i = 6000\,psi$	$r_w = 0.3\,ft$
$k_4 = 18\,mD$	$r_4 = 1055\,ft$	$q = 300\,rb/d$	
$k_5 = 40\,mD$	$r_5 = 5000\,ft$		

The procedure using equations 15, 24 and 26 is called Yeh-Agarwal method (YA). The analogous procedures obtained by using equation 28 instead of equation 26 is called the modified Yeh-Agarwal method (MYA).

Drawdown Results

Here, results to ilustrate the reliability of the procedures for estimating the permeability distribution directly from well-testing pressure data are presented. It is clear that the pressure derivative at time t is influenced by the permeability distribution at all points within a region of investigation. Thus, the instantaneous permeability can be expected to give only a rough aproximation of the actual permeability distribution when the region of investigation represents a region in wich the permeability variation is significant.

Case 1

This case pertains to a five-zone multicomposite reservoir with the following reservoir parameters:

The permeability distribution estimated with the Modified Yeh-Agarwal method is slightly closer to the correct distribution than the distribution obtained with the Yeh-Agarwal method. The results ilustrate our conclusion that the Modified Yeh-Agarwal method shows results as accurate as or slightly more accurate than the Yeh-Agarwal procedure.

References

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[3] A. J. Rosa and R. N. Horne. Reservoir description by well test analysis using cyclic flow rate variation. Dallas, 1991. SPE. Annual Technical Conference and Exhibition.

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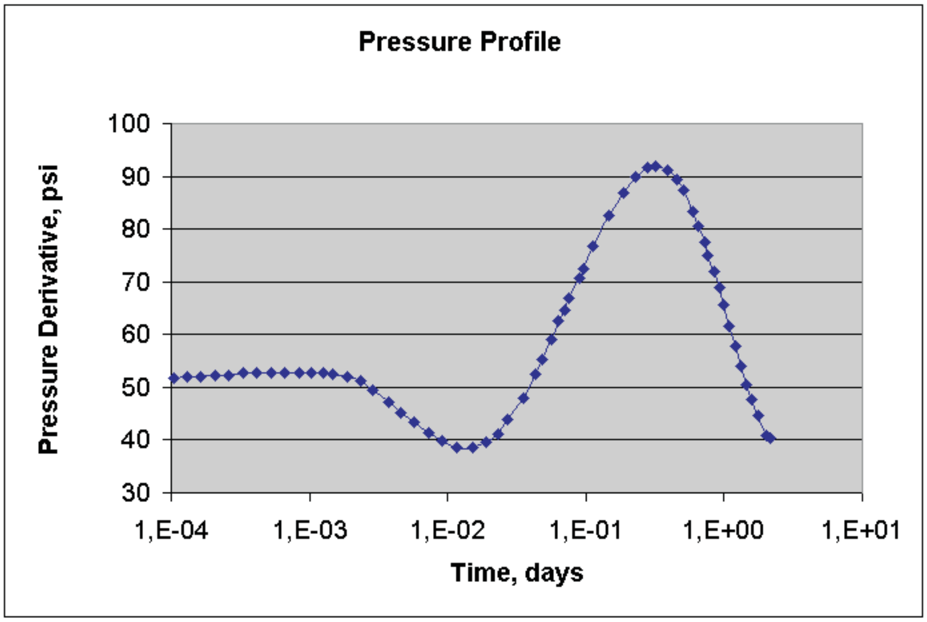

Figure 1: Pressure Derivative Profile

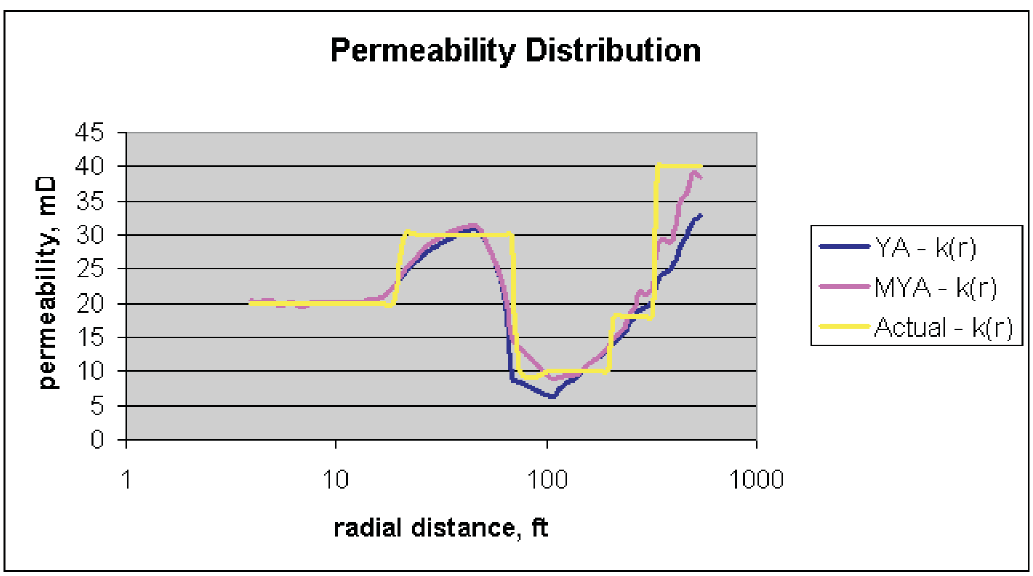

Figure 2: Permeability Profile