

## Computing time-dependent electric and magnetic fields in anisotropic media with the pulse point current

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Anisotropic materials attract a great deal of interest in modern engineering applications, in particular, micro-electronics [1, 2]. In general case a type of anisotropy specifies properties of materials, and electromagnetic waves propagation inside these materials essentially depends on the direction, generating electromagnetic fields of very peculiar forms. Modelling and simulation of waves inside these materials constitute an important interdisciplinary area of research with many cutting-edge scientific and technological applications.

Fundamental models for electromagnetic wave propagation used in science and engineering are based on the Maxwell system. In particular the mathematical models of electric and magnetic waves motions in electrically and magnetically anisotropic materials are very well known (see, for example, [1, 2, 3]). These mathematical models are related to the time-dependent Maxwell’s system containing matrices of the dielectric permittivity  $\mathcal{E}$  and magnetic permeability  $\mathcal{M}$ . For linear homogeneous non-dispersive electrically and magnetically anisotropic materials the electric and magnetic flux densities  $\mathbf{D}$  and  $\mathbf{B}$  have the following relations with the electric and magnetic fields  $\mathbf{E}$  and  $\mathbf{H}$ :  $\mathbf{D} = \mathcal{E}\mathbf{E}$ ,  $\mathbf{B} = \mathcal{M}\mathbf{H}$ , where the permittivity  $\mathcal{E}$  and the magnetic permeability  $\mathcal{M}$  are symmetric positive definite matrices. We suppose that the electric and magnetic fields  $\mathbf{E}$  and  $\mathbf{H}$  and the electric charge density  $\rho$  satisfy the following conditions

$$\mathbf{E} = 0, \quad \mathbf{H} = 0, \quad \rho = 0, \quad \text{for } t \leq 0,$$

this means there is no electric charges at the time  $t \leq 0$ ; electric and magnetic fields vanish for  $t \leq 0$ .

The Maxwell system can be written in the form of partial differential equations of the second order [4]

$$\mathcal{E} \frac{\partial^2 \mathbf{E}}{\partial t^2} + \text{curl}_x(\mathcal{M}^{-1} \text{curl}_x \mathbf{E}) = -\frac{\partial \mathbf{j}}{\partial t}, \quad (1)$$

$$\mathcal{M} \frac{\partial^2 \mathbf{H}}{\partial t^2} + \text{curl}_x(\mathcal{E}^{-1} \text{curl}_x \mathbf{H}) = \text{curl}_x(\mathcal{E}^{-1} \mathbf{j}). \quad (2)$$

Here  $x = (x_1, x_2, x_3)$  is a space variable from  $R^3$ ,  $t$  is a time variable from  $R$ ;  $\mathcal{E}^{-1}$ ,  $\mathcal{M}^{-1}$  are the inverse matrices to  $\mathcal{E}$  and  $\mathcal{M}$ , respectively;  $\mathbf{j}(x, t) = \mathbf{e} \delta(x_1) \delta(x_2) \delta(x_3) \delta(t)$ , where  $\mathbf{e} \in R^3$  is a given unit vector,  $\delta(\cdot)$  is the Dirac delta function. This form of the current density  $\mathbf{j}(x, t)$  is related to a directional impulse electric source concentrated at the origin of coordinates at the time  $t = 0$  in the direction  $\mathbf{e}$ .

The different approaches for modelling of the behavior of electric and magnetic fields may be found in the literature [3, 4, 5, 6]. The most popular is a ‘plane wave’ approach, in which the presentation of electric and magnetic fields is based on plane waves. Such presentation is a realistic approximation if, for example, a point pulse source is a great distance away from the observation point. But if a point pulse source is concentrated at a short distance from the observation point then electric and magnetic waves may take very peculiar forms. At the same time electric and magnetic fields are invisible. The simulations of these fields by modern computer tools especially for anisotropic materials allow us to see dependence between the material structure and the behavior of electric and magnetic fields.

Most of the studies and modelling electromagnetic waves had been made by numerical methods, in particular finite element method [5, 6, 7].

On the other hand nowadays computers can perform very complicated symbolic computations (in addition to numerical calculations) and this opens up new possibilities in modelling and simulation of wave propagation phenomena. Symbolic computations can be considered as a useful tool for analytical methods which can provide exact solutions of problems [8, 9, 10]. Unfortunately the exact solutions can not be found for all complex equations and systems. But when the exact solution can be found it leads to the significant simplification of modelling and simulation and allows engineers to focus on the ideas rather than on overcoming calculational difficulties.

The Cauchy problem for the Maxwell system of crystal optics with smooth data and explicit formula for an exact solution of this problem has been described by Courant in [11]. Melrose and Uhlmann in [12, 13] have constructed the microlocal parametrix for the Cauchy problem of the Maxwell system in a biaxial crystal to analyze the conical refraction, when a ray of light hits a biaxial crystal, and to explain the so-called "double ring" phenomena. Burrige and Qian in [14] have obtained a formula for a fundamental solution for the system of crystal optics (system (1), where  $\mathcal{E}$  is a diagonal matrix with the different positive elements,  $\mathcal{M}$  is the identity matrix). This formula has been obtained by the Herglotz-Petrovskii representation as an integral around loops which, for each position and time form the intersection of a plane in the space of slownesses with the slowness surface. Yakhno in [15] has used symbolic computations for the constructing an explicit formula for a fundamental solution (Green's function) for the system of crystal optics (anisotropic dielectrics) (system (1), where  $\mathcal{E}$  is a symmetric positive definite matrix with constant elements,  $\mathcal{M}$  is the identity matrix)) and then modelling and the simulation of electromagnetic waves have been obtained in [16], [17].

In this paper we present a new analytical method for constructing explicit formulae for an exact solution of an initial value problem for the Maxwell system (1), (2) with zero initial data. This method has been efficiently used for modelling electric and magnetic fields inside different electrically and magnetically anisotropic non-dispersive homogeneous materials. In general case these explicit formulae are very cumbersome and they have been computed using symbolic computations in MATLAB. We note that the simulation of the wave phenomena based on explicit formulae for exact solutions of partial differential equations is the best one according to accuracy. The robustness of our approach is based on the modern achievements of computational algebra which allow us to make symbolic transformations of cumbersome formulae and matrices. Our approach combines in a rational way analytical techniques and numerical computations.

Using this approach we have generated images of electric and magnetic fields components from a pulse pointed electric source (pulse current concentrated at the origin of coordinates at time  $t = 0$ ). Analyzing these images we have found that electric and magnetic waves in different anisotropic materials have different shapes of fronts and the different distribution of the electric and magnetic energy. Using our method we have created a collection of images of electric and magnetic fields. This collection of images is organized now as a library which contains several hundreds of images classified according to the crystal system. This library can be used as a set of patterns for study and development of anisotropic materials. At the same time it can be used for verification of new numerical methods.

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