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Harmonic modification of Prony algorithm for non-homogeneous systems of ordinary differential equations of the first order

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Let

$$\frac{d}{dt}Y(t) = AY(t) + BF(t), \quad t \in \mathbb{R} \tag{1}$$

be a system of ordinary linear differential equations with constant complex coefficients, where $Y(t)$ is unknown harmonic vector-functions of n components, $F(t)$ is known vector-function of $2m$ components, A is unknown matrix $n \times n$ with constant complex coefficients and with certain limitation on it, B is known matrix $n \times 2m$ with constant complex coefficients.

Consider the direct and inverse Cauchy task for system (1) satisfying boundary condition:

$$C_k = Y(t_0 + kd), \quad k = 0, 1, 2, \ldots, N, \quad d > 0, \quad N > r, \quad r := n + 2m, \tag{2}$$

and where t_0 is a fix time moment. Without any sacrifice for generality, we assume that $t_0 = 0$.

There exist the Prony algorithm and its modifications for solving the above task (e.g., [1-2]). In the report we are going to present in detail the conditions under which the algorithm is well posed. The received results of the research are based on the methods of solving similar task for ordinary differential equation with unknown constant coefficients ([3, chapter 2]).

Theorem. There exists a unique harmonic solution $Y(t)$ of system (1) that satisfies conditions (2), iff there is a unique vector $\vec{p} = (p_1, p_2, \ldots, p_r)$ for vectors $C_0, C_1, \ldots, C_N$ such that

$$C_{k+r} + p_1 C_{k+r-1} + \ldots + p_r C_k = 0, \quad \forall\, k = 0, 1, 2, \ldots, N - r,$$

and the polynomial $T_r(z) = z^r + p_1 z^{r-1} + \ldots + p_r$, associated with vector $\vec{p}$, has only fixed roots.

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