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On Thompson's conjecture for some simple groups with connected prime graph

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Let G be a finite group and $Z(G)$ be its center. For $x \in G$, $cl_G(x)$ denotes the conjugacy class in G containing x and $C_G(x)$ denotes the centralizer of x in G. We will use $N(G)$ to mean $\{n : G \text{ has a conjugacy class of size } n\}$. For an integer $z > 1$, we denote by $\pi(z)$ the set of prime divisors of z. The *prime graph* $GK(G)$ of a group G is the graph with vertex set $\pi(G)$ where two distinct primes r and s are joined by an edge (we write $(r, s) \in GK(G)$) if G contains an element of order rs. A list of all finite simple groups with disconnected prime graph is obtained in [5] and [7].

This talk concerns the following open conjecture of J. G. Thompson which is Problem 12.38 in the Kourovka notebook [4]:

Thompson's Conjecture: Let G be a finite group with $Z(G) = 1$. If S is a non-abelian finite simple group satisfying that $N(G) = N(S)$, then $G \cong S$.

This conjecture was posed in 1988, which is appeared in a communication letter. This conjecture has received some attention in existing literature. The most important published contributions to date can be found in a paper of Chen [2] and, a subsequent paper by Daraf-sheh [3] and the references quoted in that paper. All results arising from those papers are about the finite simple groups with disconnected prime graph. Vasiliev [6] has proved that

Thompson's conjecture holds true for $L_4(4)$ and A_{10}, which have the connected prime graph. In [1], it was shown that some non-abelian simple groups of Lie type A_n with connected prime graph satisfy in Thompson's conjecture. But Thompson's conjecture remains open for other finite simple groups with connected prime graph. The approach to Thompson's conjecture for finite simple groups with disconnected prime graph can be seen to fail for finite simple groups with connected prime graph.

In this talk, we consider the validity of Thompson's conjecture for some finite simple groups with connected prime graph.

References

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