Local finiteness conditions

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The work is dedicated to the review of some recent results on periodical groups with elements of small orders. These are the groups, which may turn out to be locally finite. And conditions which may guarantee the local finiteness of the corresponding groups are provided.

First example of such conditions may be obtained from the following theorem (received in a joint work with A.A. Maximenko):

Theorem 1. *If a group G is generated by a conjugacy class of order 3-elements whose every pair generates a subgroup that is isomorphic to one of the following groups Z_3, A_4, A_5, $Sl_2(3)$, or $SL_2(5)$, then G is locally-finite.*

Moreover, it is possible to give more details on the structure of G in this case.

Let's also consider the notion of *spectrum* $\omega(G)$, widely used in finite groups to represent the set of element orders in (periodical) group G. There is an interesting question: which $\omega(G)$ may guarantee the local finiteness of the corresponding group G?

One of the recent results obtained in joint work with V. D. Mazurov is

Theorem 2. *If G is a group, such that $\omega(G) = \{1, 2, 3, 5, 6\}$, then G is a soluble locally finite group and one of the following is true:*

1. *G is an extension of an elementary abelian 5-group by a cyclic group of order 6;*

2. *G is an extension of a nilpotent 3-group of class 3 by a dihedral group of order 10;*

3. *G is an extension of a direct product of a nilpotent 3-group of class 3 and an elementary abelian 2-group by a group of order 5.*

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Also the following result on groups of period 12 seems to be interesting and providing good insight on the structure of groups of period 12.

Statement. *Let G be a group of period 12. If G is an extension of 2-group by an element z of order 3 and G acts on a vector space over the field $GF(3)$ in such a way that every element of order 3 acts quadratically, then $[O_2(G), z]$ is nilpotent of class not greater than 2.*

Conditions in the statement are natural due to Hall-Higman theorem.