REPRESENTATION THEORY OF POLYADIC GROUPS

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A non-empty set G together with an n-ary operation $f : G^n \to G$ is called an *n-ary group* if the operation f is associative and for all $x_0, x_1, \ldots, x_n \in G$ and fixed $i \in \{1, \ldots, n\}$, there exists an element $z \in G$ such that

$$f(x_1^{i-1}, z, x_{i+1}^n) = x_0.$$

Suppose that (G, f) is an n-ary group and A is a non-empty set. We say that (G, f) *acts* on A if for all $x \in G$ and $a \in A$ corresponds a unique element $x.a \in A$ such that

(i) $f(x_1^n).a = x_1.(x_2.(x_3.\ \ldots\ .(x_n.a))\ldots)$ for all $x_1, \ldots, x_n \in G$,

(ii) for all $a \in A$, there exists $x \in G$ such that $x.a = a$,

(iii) the map $a \mapsto x.a$ is a bijection for all $x \in G$.

Suppose that an n-ary group G acts on a vector space V and we have

(1) $x.(\lambda v + u) = \lambda x.v + x.u$,

(2) $\exists p \in G\ \forall v \in V : p.v = v$.

Then we call (V, p), or simply V, a *G-module*.

The main aim of this article is to investigate the properties of these modules, with a special focus on ternary groups (the case $n = 3$). Note that, this is not the first attempt to study representations of n-ary groups, however, our method seems to be the most natural generalization of the notion of representation from binary to n-ary groups.

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