On the conjugacy problem in a group $\mathbf{F}/\mathbf{N_1} \cap \mathbf{N_2}$.

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Let F = F(A) be a free group generated by a finite alphabet A. Let N_1 (respectively N_2) be the normal closure of a finite non-empty symmetrized set R_1 (respectively R_2) of cyclically reduced words of F.

If two words u and v of F present equal elements both in F/N_1 and in F/N_2 , they do so in $F/N_1 \cap N_2$. It is natural to ask whether u and v present conjugate elements in $F/N_1 \cap N_2$, if u and v do so both in F/N_1 and in F/N_2 ? Evidently the answer is negative (the simplest example is $F = F(a, b, c), R_1 = \{a^{\pm 1}\}, R_2 = \{b^{\pm 1}\}, u = c^2ba, v = cbca\}$.

The aim of this work is to find out under what conditions on R_1 and R_2 , the solvability of the conjugacy problem in $F/N_1 \cap N_2$ follows from that in F/N_1 and F/N_2 . Here the conjugacy problem is understood in the following way: for a group $\hat{G} = F/N$, decide whether or not words u and v from F present cojugate elements in \hat{G} , and in the case of the affirmative answer find $h \in F$ such that $u = h^{-1}vh$ in \hat{G} .

It is well known (see Theorem 7.6 [1]) that if R_i satisfies the small cancellation condition C(6), then the conjugacy problem is solvable in $G_i = F/N_i$. The main result of the present paper is the following

Theorem 1 Let $R_1 \cup R_2$ be a set satisfying the small cancellation condition C(6) and $G = \langle A | R_1 \cup R_2 \rangle$ be an atorical presentation. Then the conjugacy problem is solvable in $F/N_1 \cap N_2$.

It is well known that the condition C(7) is sufficient for atoricity (the proof of this fact is similar to one of Theorem 13.3 [2]). Therefore we have:

Corollary 1 Let $R_1 \cup R_2$ be a set satisfying the small cancellation condition C(7). Then the conjugacy problem is solvable in $F/N_1 \cap N_2$.

References

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