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Two-dimensional synchronized switch schemes

Yashin A.D.

By *two-dimensional synchronized switch scheme* we mean a door with n^2 cyclic switches arranged as an $n \times n$ matrix. Every switch may be in one of m possible states. When one rotates a switch (ij) on x steps all switches in i-th row and j-th colon are rotated together on the same number of steps. The initial configuration of switches may be arbitrary. The door can be opened provided all of switches have been oriented identically. How one can open the door? We say that for given m and n this problem is *solvable* if for every initial configuration of switches there is a sequence of rotations permitting to open the door.

Theorem 1. *The problem "Open the door" is solvable iff the number m is relatively prime with numbers $n-1$ and $2n-1$.*

The problem is formalized in the ring $\mathcal{M}_n(\mathbb{Z}_m)$ of $n \times n$-matrices over the ring $\mathbb{Z}_m$ of reminders modulo m.

Let A be an initial matrix and X be a rotation matrix. X acts on A by $A \mapsto A - X + XU + UX$ where U consists on units: $(U)_{ij} = 1$ for $i, j = 1, \dots, n$.

By *the rotation operator* we mean a linear operator $\mathcal{F}$ on $\mathcal{M}_n(\mathbb{Z}_m)$ defined by $\mathcal{F}(X) \equiv -X + XU + UX$. The initial problem is reduced to the following equation

$$\mathcal{F}(X) = C \qquad (*)$$

for arbitrary matrix C.

Denote by n' and n'' inverse elements for $n-1$ and $2n-1$ respectively in the ring $\mathbb{Z}_m$ (they exist in accordance with Theorem 1). The solution of $(*)$ can be described in terms of degrees of the rotation operator.

Theorem 2. *Providing conditions of Theorem 1 the solution X of $(*)$ is of the form*

$$X = \alpha_1 C + \alpha_2 \mathcal{F}(C) + \alpha_3 \mathcal{F}^2(C),$$

where $\alpha_1 = -1 + n' + n'',\ \alpha_2 = n' + n'' - n'n'',\ \alpha_3 = -n'n''.$

e-mail: *yashin_aleksandr@list.ru*

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