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# On Engel elements in groups

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The Engel condition as natural generalization of nilpotency is very attractive for a long time and for many group-theorists. Hereby, a different types of Engel elements are studied: bounded and unbounded, left and right Engel elements. We will consider *unbounded left Engel* elements and we will call them *Engel* elements for short. Here we use definitions of Gruenberg [**3**] (see also [**4, 5**]). B.I. Plotkin calls such elements by nil-elements [**6, 7**], and he uses a term "Engel" for bounded Engel elements.

In particular cases, the set of all Engel elements constitutes a subgroup and moreover the Hirsh-Plotkin radical (see [**5, 6**]). Examples of Engel but not locally nilpotent groups are constructed by E.S. Golod [**1**] (see also [**4, 6**]). So, in general case, the set of all Engel elements may be not the Hirsh-Plotkin radical. There are analogies between Burnside problem on locally finiteness of periodical groups (or groups of finite exponent) and the problem on locally nilpotence of Engel groups (or boundary Engel groups). In this connection, a question: "will the set of all periodical elements of a group be a subgroup?" has a sufficiently simple negative answer. At the same time, an analog of this question for Engel elements:

*Does the set of all Engel elements constitute a subgroup?*

— a question raised by B.I. Plotkin [**6, 7**] was open up to now. We give a negative answer to this question:

Theorem. *There exist groups in which a product of Engel elements is not necessarily an Engel element.*

For the proof we construct a non-Engel group generated by Engel elements. In particular, we show a non-Engel element which is a product of two Engel elements. Our example is based on infinite 2-group of constructed by R.I. Grigorchuk [**2**]

Note that Plotkin's question applied to bounded Engel elements is still open. We also note that our example is a periodical 2-group, so this question is open for torsion-free group (and for groups without elements of order two). It is also interesting to study this question for totally ordered and right ordered groups.

## Список литературы

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