Мальцевские чтения
Новосибирск (2005)
Тезисы докладов

Transformations of groups. Agregatition of groups

Yu. B. Mel′nikov

In the base of the work lies author's theory of modeling. There exists a large number of the definitions of the notion "model." This is considered as a result of abstraction (for instance, in applied study), and as a result of concrete definition (for instance, in mathematical logic). When we speak about modeling, we deal with three objects: the prototype, the image and the links between them. We shall note two facts. First, the most of researchers understands the "model" as the image of prototype. Secondly, usually in the base of the definitions lies some interpretation of "similarity" between prototyping object and image object. For example, "similarity" in logic means that all theory formulas are true by any correct interpretations of variable, predicate symbol and so on. Our approach is founded on interpretation of the models as object of the certain structure. This structure includes the image of links between the prototype and its image and the image of the prototype. The structure of the image looks like the structure of model of predicate calculus. The level of "similarity" is valued by the "adequacy" — some system of the characteristics. This approach is described in our monographs [**1**, **2**] and the site http://melnikov.k66.ru. This approach allows to define correctly the operations of the algebra of the models. Let's assume, that we are required to build a model of a certain object. The building principal of new model is a complex creative work. One of the variants of the constructive decision of the problem consists in building new models with use of the system of already known models of the given object and some *operations of model algebra*. Under traditional approach the result of operations must be "similar" to prototype. In complex applied research (for instance, in ecology) source models may be "similar" to prototype in various senses, In what sense we must consider the "similarity" between the prototype and the result of operation? When we use our definition of the model this problem does not appear. The result of this operations must have required structure, and this is the single requirement. The problem consists in building a mechanism for estimation adequacy of the result of operations. This problem has the exact mathematical formalization that is an essential achievement for applied research.

We have used one of the operation of the algebra of the models for the theory of finite groups. Usually mathematical objects are considered as models, images of other objects, but now we considered groups as prototype. In this paper we shall determine *agregatition* in one particular sense which is called subset-agregatition of the group.

The universal algebra $\mathcal{H} = \langle \mathbf{H}; \mathbf{F} \rangle$ is named *the result of subset-agregatition of group* $\mathcal{G} = \langle \mathbf{G}; \{*\} \rangle$ iff $\mathbf{H} \subseteq 2^{\mathbf{G}}$ and for the operation $f \in \mathbf{F}$ and for any elements $H_1;\ H_2;\ \ldots;\ H_n \in \mathbf{H}$ we have

$$\exists h_1 \in H_1\ \ldots\ \exists h_n \in H_n \quad h_1 * h_2 * \ldots * h_n \in f\,(H_1;\ H_2;\ \ldots;\ H_n)\,.$$

We'll speak that the group $\mathcal{H}$ is the result of *subset-agregatition with fixed representatives* of group $\mathcal{G}$ iff h_i depends only from choice of H_i, i.e., there exists such a function $\varphi : \mathbf{H} \to \mathbf{G}$ that for any $H_1;\ H_2 \in \mathbf{H}$ we have $\varphi(H_1) * \varphi(H_2) \in H_1 \circ H_2$. If $\varphi(H_1) * \varphi(H_2) * \ldots * \varphi(H_k) \in H_1 \circ H_2 \circ \ldots \circ H_k$ for any k, then we have the *subset-agregatition with fixed strong representatives.* If $\forall H_1, H_2 \in \mathbf{H} \quad H_1 \neq H_2 \Rightarrow H_1 \cap H_2 = \varnothing$ then this subset-agregatition is named *excluding subset-agregatition.*

Теорема 1. *For any finite universal algebra $\mathcal{A} = \langle \mathbf{A}; \mathbf{F} \rangle$ there exists such a finite cyclic group $\mathcal{Z} = \langle \mathbf{Z}; \{*\} \rangle$ that $\mathcal{A}$ is isomorphic to some result of subset-agregatition of the group $\mathcal{Z}$.*

Теорема 2. *For any finite group $\mathcal{H} = \langle \mathbf{H}; \{\circ\} \rangle$ there exists such finite cyclic group $\mathcal{Z} = \langle \mathbf{Z}; \{*\} \rangle$ (elementary abelian group $\mathcal{P} = \langle \mathbf{P}; \{*\} \rangle$), that $\mathcal{H}$ is isomorphic to some result of subset-agregatition with fixed representatives of the group $\mathcal{Z}$ (the group $\mathcal{P}$).*

Теорема 3. *If group $\mathcal{H}$ can be presented as excluding subset-agregatition abelian group $\mathcal{G}$ with fixed representatives, than group $\mathcal{H}$ also is abelian.*

Теорема 4. *Let finite group $\mathcal{H} = \langle \mathbf{H}; \{\circ\} \rangle$ be the result of excluding subset-agregatition with fixed strong representative of the finite group $\mathcal{G}$, and group $\mathcal{S}$ is induced in $\mathcal{G}$ by set $\mathbf{S} = \Big\{\varphi(H_1) * \ldots * \varphi(H_m) \,\Big|\, H_i \in \mathbf{H};\ m \in \mathbb{N}\Big\}$. Then there exists normal subgroup $\mathcal{K} = \langle \mathbf{K}; \{\bullet\} \rangle$ of the group $\mathcal{S}$ such that, for any $H_0 \in \mathbf{H}$ we have $H_0 \cap \mathbf{S} \subseteq (\varphi(H_0) * \mathbf{K})$.*

In other word, result of excluding subset-agregatition with fixed strong representatives practically does not differ from a factor-group.

Список литературы

[1] Мельников Ю.Б. Математическое моделирование: структура, алгебра моделей, обучение построению математических моделей: Монография.– Екатеринбург: Уральское издательство, 2004, 384 с.

[2] Мельников Б.Н., Мельников Ю.Б. Геотехногенные структуры: теория и практика: Монография. Екатеринбург. Уральское изд-во, 2004. 556 с.

Ural State Pedagogical University
E-mail: melnikov@k66.ru