

## Large characteristic subgroups and ideals satisfying multilinear commutator identities

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We prove that if a group  $G$  has a subgroup  $H$  of finite index  $n$  satisfying a multilinear commutator identity  $\kappa(H) = 1$ , then  $G$  has also a characteristic subgroup  $C$  satisfying the same identity  $\kappa(C) = 1$  and having finite index bounded in terms of  $n$  and the weight of  $\kappa$ . Earlier such a result was known in folklore only for abelian subgroups. As an illustration, our result is applied to finite groups with an almost regular automorphism of order 4 to obtain a subgroup of bounded index with a 'strong' bound for the nilpotency class of its derived subgroup, which gives a positive solution to Shumyatsky's Problem 11.126 in Kurovka Notebook.

We prove that if a Lie algebra  $L$  has a nilpotent ideal of nilpotency class  $c$  and of finite codimension  $r$ , then  $L$  has also a characteristic (that is, invariant under all automorphisms) nilpotent ideal of class  $\leq c$  and of finite codimension bounded in terms of  $r$  and  $c$ . We also prove a similar result for groups, where the role of dimension is played by rank: if a group  $G$  has a normal nilpotent subgroup  $H$  of class  $c$  such that the quotient group  $G/H$  has finite rank  $r$  and either  $G$  is torsion-free or  $H$  is periodic, then  $G$  has also a characteristic nilpotent subgroup  $C$  of class  $\leq c$  with quotient  $G/C$  of finite rank bounded in terms of  $r$  and  $c$ .

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Joint results with N. Yu. Makarenko.

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