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Abstracts

Primitive normal polygons

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In this work we investigate the polygons with primitive normal theory. Remind the definition of primitive normal theory.

Let T be a complete theory of language L and $\mathfrak{C}$ be a rather large and saturate model of the theory T. The formula of a form

$$\exists x_1 \cdots \exists x_n (\Phi_0 \wedge \cdots \wedge \Phi_k)$$

is called primitive, where Φ_i , $i \leq k$, are atomic formulae. The theory T is called primitive normal if either $\Phi(\mathcal{C}, \bar{a}) = \Phi(\mathcal{C}, \bar{b})$ or $\Phi(\mathcal{C}, \bar{a}) \cap \Phi(\mathcal{C}, \bar{b}) = \emptyset$ for any primitive formula $\Phi(\bar{x}, \bar{y})$ and any n–corteges $\bar{a}, \bar{b}$ of elements from $\mathfrak{C}$.

It is well known that the theory of any module is primitive normal. Moreover for any structure N if the theory of Cartesian degree $Th(N^\omega)$ is stable than the theory $Th(N)$ is primitive normal. The additive and primitive connected theories are primitive normal [1,2].

For a monoid S a (left) S-polygon ${}_SA$ is a set A on which S acts unitarity from the left in usual way. In [3] it is proved that the theories of all S–polygons are primitive normal iff S is linearly ordered monoid, that is $(\{Sa \mid a \in S\}, \subseteq)$ is linearly ordered set. Here it is given the characterization of S–polygons with primitive normal theory.

For the formulation the theorem it is necessary to give some notions. The equivalence α on some set X of n-corteges of elements from $\mathfrak{C}$ defined in $\mathfrak{C}$ by some primitive formula $\Phi(\bar{x}_1, \bar{x}_2)$ is called primitive equivalence. The equivalences α β on the set X of n–corteges of elements from $\mathfrak{C}$ are called permutation if $\alpha \circ \beta = \beta \circ \alpha$.

Theorem. For any S–polygon ${}_SA$ the theory $Th({}_SA)$ is primitive normal iff any two primitive equivalents on A are permutation.

References

[1] Palyutin E.A. Additive theories, in: Proceedings of Logic Colloquim'98 (Lectures Notes in Logic, 13), ASl, Massachusetts, 2000, 352–356.

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