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Three Computability Results on the Decomposability Property of Theories

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Definition 1. Let $\mathcal{T}$ be a theory in a signature Σ. We call $\mathcal{T}$ **decomposable**, if there exist theories $\mathcal{S}_1, \mathcal{S}_2$ with disjoint signatures $\Sigma_1 \cap \Sigma_2 = \varnothing$, $\Sigma_1 \neq \varnothing \neq \Sigma_2$, $\Sigma_1 \cup \Sigma_2 = \Sigma$ such that $\mathcal{T} \sim \mathcal{S}_1 \cup \mathcal{S}_2$. The theories $\mathcal{S}_1, \mathcal{S}_2$ are called **decomposition components** of $\mathcal{T}$.

The decomposability notion was first formulated in [**1**] and the study of this property was set in [**2**]. The ability to recognize decomposable theories and find their decomposition components proves useful in many applications of automated reasoning. It allows for distributed processing of large declarative knowledge bases and serves as a basis for component-based approach to their development [**3, 4**]. We must note that *modularization* of theories also becomes crucial for reducing search space and using multiple reasoners in automated theorem proving [**5**]. Hence, it is important to understand computability limits of the related tasks and, among them, of recognizing decomposability. Here we present first results on computability of this property.

Definition 2. The **decomposability problem** for finite elementary theories in a signature Σ is the set of indices of decomposable finite elementary theories in Σ.

Theorem 1. *The decomposability problem for finite elementary theories of a finite signature is computably enumerable. Besides, there exists a finite signature Σ such that the decomposability problem for finite elementary theories in Σ is Σ^0_1–complete.*

Definition 3. A theory $\mathcal{T}$ is called **universal Horn theory**, if every sentence of $\mathcal{T}$ is a quasi-equality.

Theorem 2. *There exists a finite signature Σ such that the decomposability problem for finite universal Horn theories in Σ is Σ^0_1–complete and thus, undecidable.*

Consider a signature $\sigma = \{P_0, P_1, \ldots; C_0, C_1, \ldots\}$, where $P_i,\ i \in \omega$ are unary predicates and $C_j,\ j \in \omega$ are constants.

Theorem 3. *There exists an algorithm to determine whether a given finite set of sentences in the signature σ is decomposable.*

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