Characterization of alternating and symmetric groups by spectrum

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- G is a finite group
- $\pi(G)$ is the set of prime divisors of $|G|$
- $\omega(G)$ is the spectrum of G, that is the set of its element orders
- G and H are isospectral if $\omega(G) = \omega(H)$

$h(G)$ is the number of pairwise non-isomorphic finite groups with the spectrum equal to $\omega(G)$.

- G is called recognizable if $h(G) = 1$,
- almost recognizable if $h(G) < \infty$,
- nonrecognizable if $h(G) = \infty$

Shi, 1984

The group A_5 is recognizable.

Brandl, Shi, 1991

The group A_6 is nonrecognizable.

Mazurov, Zavarnitsine, 1999

Suppose that the factor group $H = G/N$ of a finite group G is isomorphic to a symmetric or alternating group of degree m, where $m \geq 6, m \neq 8$ and $N \neq 1$. Then $\omega(G) \neq \omega(H)$.

Kondrat'ev, Mazurov, 2000; Zavarnitsine, 2000

The group A_n is recognizable if $n = p, p+1, p+2$, where $p \geq 7$ is a prime.

Mazurov, 1998

The group A_{10} is nonrecognizable.

Zavarnitsine, 2000

The group A_{16} is recognizable.

Shao, Jiang, 2010

The group A_{22} is recognizable.

The group A_n is recognizable if $5 \leq n \leq 25$ and $n \neq 6, 10$.

pr(n) the largest prime not exceeding n.

Vakula, 2010

Let n be an integer greater than 21 and G be a group isospectral to the group A_n. Then, every chief series of the group G possesses a factor isomorphic to the group A_k for some k from the interval $[pr(n), n]$. Moreover, no other factor of this series contains the element $pr(n)$ in its spectrum.

Gorshkov, 2012

Suppose G is a finite group with $\omega(G) = \omega(A_n)$, where $n \geq 5$, $n \neq 6, 10$. Then G is isomorphic to A_n.

Brandl, Praeger, Shi, Mazurov, Darafsheh, Modhaddamfar, Zavarintsine, 1991 – 2001

The group S_n is recognizable if $n = 9, 12, 14$, or n is a prime greater than 5.
The group S_n is nonrecognizable if $n = 5, 6, 8$.

Gorshkov, 2013

Suppose G is a finite group with $\omega(G) = \omega(S_n)$, where $n \geq 5$, $n \neq 5, 6, 8, 10, 15, 16, 18, 21, 25, 27, 33, 35, 39, 45$. Then G is isomorphic to S_n.

Thank you for your attention!