DYNAMIC MEREOTOPOLOGY III. WHITEHEADEAN TYPE OF INTEGRATED POINT-FREE THEORIES OF SPACE AND TIME

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1. Whitehead's view on point-free theory of space and time
   - Whitehead's theory of space
   - Whitehead's theory of time
2. Some facts for contact and precontact algebras
   - Contact and precontact algebras
3. Standard (point-based) Dynamic Contact Algebras
   - Time structures
   - Point based Dynamic model of space
   - Time conditions and time axioms
   - Time representatives
4. Dynamic Contact Algebras
   - Abstract definition
   - Representation theory for DCA-s
5. Concluding remarks

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Alfred North Whitehead (15.02.1861 - 30.12.1947)

- **Alfred North Whitehead** (February 15, 1861 - December 30, 1947) was an English **mathematician** and **philosopher**, known as a founder of the contemporary **process philosophy.**
- He is well-known also as the co-author with Bertrand Russell of the famous book "**Principia Mathematica**".
- They intended to write a special part of the book related to the **foundation of geometry**, but due to some disagreements between them this part had not been written.
- Later on Whitehead formulated his own **program for a new theory of space and time.**

  The best articulation of this program is in this quote from his book **The Organization of Thought (1917)** (page 195):

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A quote from Whitehead

"...It follows from the relative theory that **a point should be definable in terms of the relations between material things.** So far as I am aware, this outcome of the theory has escaped the notice of mathematicians, who have invariably assumed the point as the ultimate starting ground of their reasoning....
Similar explanations apply to time.
Before the theories of space and time have been carried to a satisfactory conclusion on the relational basis, a long and careful scrutiny of the definitions of points of space and instants of time will have to be undertaken, and many ways of effecting these definitions will have to be tried and compared.
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Comments

A more detailed program of how to build the **theory of space** based on the above ideas was described in his famous book

A. N. Whitehead, **Process and Reality**, New York, MacMillan, 1929.

Outline

Whitehead's theory of space

The theory of space

based on the Whiteheadean approach is known now as **Region Based Theory of Space (RBTS)**.

- Unlike the classical Euclidean approach, based on the primitive notions **point, line** and **plane**, which are abstract features having no existence in reality as separate things,
- RBTS is based on a more realistic primitive notions like **region** and some spatial relations between regions like **part-of** and **contact**.

Some terminology:

- **RBTS=MEREOTOPOLOGY**
- **MEREOLOGY=theory of parts and wholes**, basic notions: regions, part-of relation, overlap, underlap,
- TARSKI: MEREOLOGY ≅ Boolean algebra,
- **MEREOTOPOLOGY=MEREOLOGY + TOPOLOGY**, new basic relation: **contact** relation,
- **CONTACT ALGEBRA**= **Boolean algebra + contact** ≅ MEREOTOPOLOGY,
- standard models of contact algebras are regular closed subsets of certain topological spaces and two regions are in a contact if they have a common point

Outline

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Whitehead's theory of time

was developed mainly in his books

A. N. Whitehead,**Science and the Modern World.** New Work, MacMillan , 1925.
and
Process and Reality
under the name **Epochal Theory of Time (ETT).**

Whitehead claims that the theory of time can not be separated from the theory of space and have to be extracted from the existing things in reality and some of their spatio-temporal relations between them considered as primitives. For instance, like points, moments of time do not have separate existence in reality and consequently have to be defined on the base of the primitives of the theory.

Whitehead's theory of time

- Unfortunately, unlike his program how to build mathematical theory of space given in [**Proces and Reality**], Whitehead did not describe analogous program for his theory of time.
- Whitehead introduced and analyzed many notions related to ETT but mainly in an informal way, which fact makes extremely difficult to obtain clear mathematical theory corresponding to ETT.
- So, any such attempt will be only an approximate partial formalization of Whitehead's ideas and the claim that it corresponds to what Whitehead had in mind always will be disputable.

THE AIM OF THE TALK

In this talk we will try to make the next step in the realization of Whitehead's program for an integrated point-free theory of space and time. The algebraic counterpart of the theory is the notion of DYNAMIC CONTACT ALGEBRA

Like the equation **Contact algebra=mereotopology**, we introduce the new equation:

DYNAMIC CONTACT ALGEBRA=DYNAMIC MEREOTOPOLOGY

This terminology motivates the title of the paper, which is a third in a series of papers devoted to the subject and in a sense it extends and updates both of them.

Outline

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Contact and precontact algebras

Definition

Let $\underline{B} = (B, 0, 1, ., +, *)$ be a non-degenerate Boolean algebra and C – a binary relation in B. C is called a **contact** relation in $\underline{B}$ if it satisfies the axioms

$(C1)$ If xCy, then $x, y \neq 0$,
$(C2)$ If xCy and $x \leq x'$ and $y \leq y'$, then $x'Cy'4$,
$(C3')$ If $xC(y+z)$, then xCy or xCz,
$(C3'')$ If $(x+y)Cz$ then xCz or yCz,
$(C4)$ If $x.y \neq 0$, then xCy
$(C5)$ If xCy, then yCx.

If C is a contact relation in $\underline{B}$ then the pair (B, C) is called a **contact algebra**, CA.

If we omit the axioms $(C4)$ and $(C5)$ then C is called a precontact relation and the pair (B, C) - a **precontact algebra**

Mereological relations and ontological existance

Definitions of basic mereological relations between regions:

- **part-of:** $a \leq b$ – just the lattice ordering of B
- **overlap:** aOb iff $a.b \neq 0$.
- **underlap (dual overlap)** xUy iff $x + y \neq 1$
- **predicate of ontological existance** x exists iff $x \neq 0$

Comments.

Topological example of CA

The CA of regular closed sets

Let X be an arbitrary topological space. A subset a of X is *regular closed* if $a = Cl(Int(a))$. The set of all regular closed subsets of X will be denoted by $RC(X)$. It is a well-known fact that regular closed sets with the operations

- $a + b = a \cup b$,
- $a.b = Cl(Int(a \cap b))$,
- $a^* = Cl(X \setminus a)$, and
- $0 = \varnothing$, $1 = X$
  form a Boolean algebra. If we define **contact** by
- aCb iff $a \cap b \neq \varnothing$, then $RC(X)$ becomes a contact algebra.

Topological representation of CA

Theorem

Topological representation theorem for contact algebras. *For every contact algebra* (B, C) *there exists a topological space* X *and an embedding* h *into the contact algebra* $RC(X)$.

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Clans

The abstract points in contact algebra are called clans (the name "clan" is taken from the theory of proximity spaces)

Definition

Let (B, C) be a contact algebra. A subset $\Gamma \subseteq B$ is called a **clan** if the following conditions are satisfied:

- $1 \in \Gamma$ and $0 \notin \Gamma$,
- If $a \in \Gamma$ and $a \leq b$ then $b \in \Gamma$,
- If $a + b \in \Gamma$ then $a \in \Gamma$ or $b \in \Gamma$,
- If $a, b \in \Gamma$ then aCb.
  Clans which are maximal with respect inclusion are called **maximal clans.**

Every clan can be extended into a maximal clan.

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  Clans which are maximal with respect inclusion are called **maximal clans.**

Every clan can be extended into a maximal clan.

Every ultrafilter is a clan but there are clans which are not ultrafilters.

General construction of clans

- Define in the set of ultrafilters the relation $U_1 \rho U_2$ iff $U_1 \times U_2 \subseteq C$.
- Let U_i, $i \in I$ be a nonempty family of ultrafilters such that every two are in the relation ρ. Then $\Gamma = \bigcup_{i \in I} U_i$ is a clan and every clan can be represented in this form.
- The set of all clans of $\underline{B}$ is denoted by $CLANS(\underline{B})$.
- For $a \in B$ we denote by $h(a) = \{\Gamma \in CLANS(\underline{B}) : a \in \Gamma\}$.
- The set $CLANS(\underline{B})$ with the set $\{h(a) : a \in B\}$ as a closed base define a topological space and h is an embedding of $\underline{B}$ into the contact algebra $RC(CLANS(\underline{B}))$ required in the Representation theorem for contact algebras.

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Clusters

Definition

Let (B, C) be a contact algebra. A subset $\Gamma \subseteq B$ is called a **cluster** in (B, C) if it is a clan and for every $x \notin \Gamma$ there exists an $y \in \Gamma$ such that $x\overline{C}y$.

Clusters will be used later on as abstract **time points.** The name is adopted from the theory of proximity spaces.

Lemma

Let (B, C) be a contact algebra satisfying the following axiom (called Efremovich axiom)
(CE) $x\overline{C}y \rightarrow (\exists z \in B)(x\overline{C}z$ and $z^*\overline{C}y)$
Then Γ is a cluster in (B, C) iff Γ is a maximal clan in (B, C).

Examples of precontact algebras

Lemma

Let X be a non-empty set and R be a binary relation in X. Let $\mathbf{B}(X)$ be the Boolean algebra of all subsets of X. For $a, b \in \mathbf{B}(X)$ define aC_Rb iff $(\exists x \in a)(\exists y \in b)(xRy)$. Then C_R is a precontact relation in $\mathbf{B}(X)$. More over:

- *R is reflexive relation iff C_R satisfies the axiom (C4) $a.b \neq 0 \rightarrow aCb$,*
- *R is symmetric relation iff C_R satisfies the axiom (C5) $aC_Rb \rightarrow bC_Ra$,*
- *R is transitive relation iff C_R satisfies the Efremovich axiom (CE) $x\overline{C}y \rightarrow (\exists z \in B)(x\overline{C}z$ and $z^*\overline{C}y)$*

Outline

Time structures

By a time structure we mean any relational system $(T, \prec)$, where T is a nonempty set of **points of time** and $\prec$ is a binary relation in T called **before-after** relation (or **time order**). The relation $\prec$ may satisfy some of the following axioms called **properties of time:**

- **(RS)** *Right seriality* $(\forall m)(\exists n)(m \prec n)$,
- **(LS)***Left seriality* $(\forall m)(\exists n)(n \prec m)$,
- **(Up Dir)** *Updirectedness* $(\forall i, j)(\exists k)(i \prec k$ and $j \prec k)$,
- **(Down Dir)** *Downdirectedness* $(\forall i, j)(\exists k)(k \prec i$ and $k \prec j)$,
- **(Dens)** *Density* $i \prec j \rightarrow (\exists k)(i \prec k$ and $k \prec j)$,
- **(Ref)** *Reflexivity* $(\forall m)(m \prec m)$,
- **(Irr)** *Irreflexivity* $(\forall m)($ not $m \prec m)$,
- **(Lin)** *Linearity* $(\forall m, n)(m \prec n$ or $n \prec m)$,
- **(Tri)** *Trichotomy* $(\forall m, n)(m = n$ or $m \prec n$ or $n \prec m)$,

Outline

Point based dynamic model of space. "Snapshot" construction

Suppose that we want to describe a dynamic environment consisting of a regions changing in time.

- First we suppose that we are given a time structure $\underline{T} = (T, \prec)$ and want to know what is the spatial configuration of regions at each moment of time $m \in T$.
- We assume that for each $m \in T$ the spatial configuration of the regions forms a contact algebra $(\underline{B}_m, C_m)$, (**coordinate contact algebra**) which is considered as a "snapshot" of this configuration.
- We identify a given changing region a with the series $< a_m >_{m \in T}$ of snapshots and call such a series a **dynamic region**. We denote by $B(\underline{T})$ the set of all dynamic regions.

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"Snapshot" construction

- If $a =< a_m >_{m \in T}$ is a given dynamic region then a_m can be considered as a **at the time point** m,
- $a_m \neq 0_m$ means that a **exists at the time point** m,
- $a_m C_m b_m$ means that a **and** b **are in a contact at the moment** m.
- We assume that the set $B(\underline{T})$ is a Boolean algebra, i.e. a mereology with Boolean constants and operations defined as follows:
- $1 =< 1_m >_{m \in T}$, $0 =< 0_m >_{m \in T}$,
- Boolean ordering $a \leq b$ iff $(\forall m \in T)(a_m \leq_m b_m)$ and
- Boolean operations are defined "coordinatewise": $a + b =_{def} < a_m(+_m)b_m >_{m \in T}$, $a.b =_{def} < a_m(._m)b_m >_{m \in T}$, $a^* =_{def} < a^*_m >_{m \in T}$.

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- If $a =< a_m >_{m \in T}$ is a given dynamic region then a_m can be considered as *a* **at the time point** *m*,
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"Snapshot" construction

- Note that the Boolean algebra $B(\underline{T})$ is a subalgebra of the Cartesian product $\prod_{m\in T} B_m$ of the contact algebras $(\underline{B}_m, C_m)$, $m \in T$.
- A model which coincides with the Cartesian product is called a **full model.**
- $B(\underline{T})$ is called a **rich model** if it contains all dynamic regions a such that for all $m \in T$ we have $a_m = 0_m$ or $a_m = 1_m$.
  Obviously full models are rich.

Basic spatio-temporal relations

Space contact $aC^s b$ iff $(\exists m \in T)(a_m C_m b_m)$.

Time contact $aC^t b$ iff $(\exists m \in T)(a_m \neq 0_m$ and $b_m \neq 0_m)$.
It can be considered also as a kind of **simultaneity relation** or **contemporaneity** relation studied in Whitehead's works. This suggests to call a and b **contemporaries** if $aC^t b$.

Precedence $a\mathcal{B}b$ iff $(\exists m, n \in T)(m \prec n$ and $a_m \neq 0_m$ and $b_n \neq 0_n)$.
This relation is mentioned by Whitehead without explicit formal definition.

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Standard Dynamic Contact Algebra

Definition

Dynamic model of space $B(\underline{T})$ supplied with the relations C^s, C^t, and $\mathcal{B}$ is called a **standard dynamic contact algebra**, standard DCA for short. It is called **rich** if the dynamic model of space is rich and **full** if the dynamic model of space is full.

Lemma

(i) C^s is a contact relation,
(ii) C^t is a contact relation satisfying the following additional condition
$(C^s \to C^t)$ $aC^s b \to aC^t b$.
If the algebra is rich then C^t satisfies the Efremovich axiom
$(C^t E)$ *If* $a\overline{C}^t b$, *then there exists* c *such that* $a\overline{C}^t c$ *and* $c^* \overline{C}^t b$
(iii) $\mathcal{B}$ is a precontact relation.

Outline

A correlation between abstract properties of time structures and time axioms in standard DCA

- **(RS)** *Right seriality* $(\forall m)(\exists n)(m \prec n) \Longleftrightarrow$
  (rs) $a \neq 0 \rightarrow a\mathcal{B}1$,
- **(LS)***Left seriality* $(\forall m)(\exists n)(n \prec m) \Longleftrightarrow$
  (ls) $a \neq 0 \rightarrow 1\mathcal{B}a$,
- **(Up Dir)** *Updirectedness* $(\forall i, j)(\exists k)(i \prec k$ and $j \prec k) \Longleftrightarrow$
  (up dir) $a \neq 0 \wedge b \neq 0 \rightarrow a\mathcal{B}p \vee b\mathcal{B}p^*$,
- **(Down Dir)** *Downdirectedness* $(\forall i, j)(\exists k)(k \prec i$ and $k \prec j)$
  $\Longleftrightarrow$
  (down dir) $a \neq 0 \wedge b \neq 0 \rightarrow p\mathcal{B}a \vee p^*\mathcal{B}b$,
- **(Dens)** *Density* $i \prec j \rightarrow (\exists k)(i \prec k \wedge k \prec j) \Longleftrightarrow$
  (dens) $a\mathcal{B}b \rightarrow a\mathcal{B}p$ or $p^*\mathcal{B}b$,

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- **(Ref)** *Reflexivity* $(\forall m)(m \prec m) \Longleftrightarrow$
  (ref) $aC^t b \rightarrow a\mathcal{B}b$,
- **(Irr)** *Irreflexivity* $(\forall m)($ not $m \prec m) \Longleftrightarrow$
  (irr) $a\mathcal{B}b \rightarrow (\exists c, d)(c\mathcal{B}d$ and $aC^t c$ and $bC^t d$ and $c\overline{C}^t d)$,
- **(Lin)** *Linearity* $(\forall m, n)(m \prec n \vee n \prec m) \Longleftrightarrow$
  (lin) $a \neq 0 \wedge b \neq 0 \rightarrow a\mathcal{B}b \vee b\mathcal{B}a$,
- **(Tri)** *Trichotomy* $(\forall m, n)(m = n$ or $m \prec n$ or $n \prec m) \Longleftrightarrow$
  (tri) $(aC^t c$ and $bC^t d$ and $c\overline{C}^t d) \rightarrow (a\mathcal{B}b$ or $b\mathcal{B}a)$,
- **(Tr)** *Transitivity* $i \prec j$ and $j \prec k \rightarrow i \prec k \Longleftrightarrow$
  (tr) $a\overline{\mathcal{B}}b \rightarrow (\exists c)(a\overline{\mathcal{B}}c \wedge c^*\overline{\mathcal{B}}b)$.

The conditions denoted by (xxx) using small letters are called **time axioms**

Outline

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Time representatives

Consider the phrases:

- "the epoch of Leonardo",
- "the epoch of Renaissance",
- "the geological age of the dinosaurs",
- "the time of the First World War".
- All these phrases indicate a concrete unit of time named by something which happened or existed at that time and not in some other time.
  These examples suggest to introduce in the standard dynamic contact algebras a special set of dynamic regions called **time representatives**, which are regions existing at a unique time point.

Time representatives.Formal definition

Definition

A region c in a standard DCA is called a **time representative** if there exists a time point $i \in T$ such that $c_i \neq 0_i$ and for all $j \neq i$, $c_j = 0_j$. We say also that c is a representative of the time point i and indicate this by writing $c = c(i)$. In the case when $c_i = 1_i$, c is called **universal time representative**. We denote by **TR** (**UTR**) the set of (universal) time representatives in a given standard DCA.

Lemma

Abstract properties of time representatives. *Let $B(\underline{T})$ be a rich standard DCA. Then the following conditions for time representatives are true:*

(TR1) $c \in TR$ iff $c \neq 0$ and $(\forall a, b)(aC^t c$ and $bC^t c \rightarrow aC^t b)$.

(TR2) $c \in UTR$ iff $c \in TR$ and $c\overline{C}^t c^*$

(TRC^t) If $aC^t b$, then $(\exists c \in UTR)(aC^t c$ and $bC^t c)$.

(TRC^s) If $aC^s b$ then $(\exists c \in UTR)((a.c)C^s b)$.

(TR$\mathcal{B}$1) If $c \in TR$, $c\mathcal{B}b$ and $aC^t c$, then $a\mathcal{B}b$.

(TR$\mathcal{B}$2) If $d \in TR$, $a\mathcal{B}d$ and $bC^t d$, then $a\mathcal{B}b$.

(TR$\mathcal{B}$3) If $a\mathcal{B}b$, then $\exists c \in UTR$ such that $c\mathcal{B}b$ and $aC^t c$.

(TR$\mathcal{B}$4) If $a\mathcal{B}b$, then $\exists d \in UTR$ such that $a\mathcal{B}d$ and $bC^t d$.

Abstract properties of time representatives

Lemma

Let $B(\underline{T})$ be a rich standard DCA with time structure $(T, \prec)$ and let $c(i)$ and $c(j)$ be the universal time representatives for the time points i and j respectively. Then :

(UTR$\mathcal{B}$11) $(\forall p \in B)(p\mathcal{B}c(i)$ *or* $p^*\mathcal{B}c(j))$ *iff* $(\exists c(k) \in UTR)(c(k)\mathcal{B}c(i)$ *and* $c(k)\mathcal{B}c(j))$.

(UTR$\mathcal{B}$12) $(\forall p \in B)(p\mathcal{B}c(i)$ *or* $c(j)\mathcal{B}p^*)$ *iff* $(\exists c(k) \in UTR)(c(k)\mathcal{B}c(i)$ *and* $c(j)\mathcal{B}c(k))$.

(UTR$\mathcal{B}$21) $(\forall p \in B)(c(i)\mathcal{B}p$ *or* $p^*\mathcal{B}p^*)$ *iff* $(\exists c(k) \in UTR)(c(i)\mathcal{B}c(k)$ *and* $c(k)\mathcal{B}c(j))$.

(UTR$\mathcal{B}$22) $(\forall p \in B)(c(i)\mathcal{B}p$ *or* $c(j)\mathcal{B}p^*)$ *iff* $(\exists c(k) \in UTR)(c(i)\mathcal{B}c(k)$ *and* $c(j)\mathcal{B}c(k))$.

NOW, the present time

Whitehead very often is talking in **PR** about
Present epoch, *Present cosmic epoch*, *Contemporary World*, *Actual World*
considering all these phrases as synonyms. In the common language "Present epoch" is just the state of all things which exist "now".
To represent the *present epoch* we introduce a special time representative named **NOW** and the point of time representing by **NOW** is denoted by **now**.
Considering **now** requires to extend the signature of time structure - $(T, \prec, \textbf{now})$ assuming $\textbf{now} \in T$.

Some definable notions by NOW

- a **exists now** $\Leftrightarrow_{def}$ aC^tNOW,
- a **exists sometimes in the future** $\Leftrightarrow_{def}$ $NOW\mathcal{B}a$,
- a **exists always in the future** $\Leftrightarrow_{def}$ $(\forall b \in TR)(NOW\mathcal{B}b \rightarrow aC^tb)$,
- a **exists always** $\Leftrightarrow_{def}$ $(\forall b \in TR)(aC^tb)$,
- a **exists sometimes in the past** $\Leftrightarrow_{def}$ $NOW\mathcal{B}a$,
- a **exists always in the past** $\Leftrightarrow_{def}$ $(\forall b \in TR)((b\mathcal{B}NOW \rightarrow aC^tb)$.

Some definable notions by NOW

For the next definitions we assume that $NOW \in UTR$.

- a **is always in a contact with** b (in symbols $aC^{\forall}b$) $\Leftrightarrow_{def}$ $(\forall c \in UTR)((a.c)C^s(b.c)$
- a **is in a contact with** b **now** $\Leftrightarrow_{def}$ $(a.NOW)C^s(b.NOW)$,
- a **is in a contact with** b **sometimes in the future** $\Leftrightarrow_{def}$ $(\exists c \in UTR)((a.c)C^s(b.c)$ and $NOW\mathcal{B}, c)$,
- a **is in a contact with** b **always in the future** $\Leftrightarrow_{def}$ $(\forall c \in UTR)(NOW\mathcal{B}c \rightarrow (a.c)C^s(b.c))$,
- a **is in a contact with** b **sometimes in the past** $\Leftrightarrow_{def}$ $(\exists c \in UTR)((a.c)C^s(b.c)$ and $a\mathcal{B}NOW$ and $b\mathcal{B}NOW)$,
- a **is in a contact with** b **always in the past** $\Leftrightarrow_{def}$ $(\forall c \in UTR)(c\mathcal{B}NOW \rightarrow (a.c)C^s(b.c))$.

Outline

By a **dynamic contact algebra** (DCA for short) we mean any system $\underline{B} = (B, 0, 1, ., +, ^*, C^s, C^t, \mathcal{B}, TR, UTR, NOW)$ where $(B, 0, 1, ., +, ^*)$ is a non-degenerate Boolean algebra and:

(i) C^s is a contact relation on B called *space contact*,

(ii) C^t is a contact relation on B, called *time contact* satisfying the following additional axioms:

$(C^s \to C^t)$ $aC^sb \to aC^tb$,

(C^tE) the Efremovich axiom for C^t,

(iii) $\mathcal{B}$ is a precontact relation, called **precedence relation**.

(iv) TR - **time representatives**, and UTR - **universal time representatives**, are subsets of B satisfying the following axioms (TR1), (TR2), (TRC^t), (TRC^s), (TR$\mathcal{B}$1), (TR$\mathcal{B}$2), (TR$\mathcal{B}$3), (TR$\mathcal{B}$4), (UTR$\mathcal{B}$11), (UTR$\mathcal{B}$12), (UTR$\mathcal{B}$21), (UTR$\mathcal{B}$22).

(v) $NOW \in UTR$.

We consider also DCA-s satisfying some of the **time axioms** **(rs), (ls), (up dir), (down dir), (dens), (ref), (irr), (lin), (tri), (tr**

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UTR-clusters as abstract time points

Custers in a DCA with respect to the time contact C^t are called t-clusters.

Definition

UTR-clusters. Let $\underline{B}$ be a DCA and Γ be a t-cluster in B. Γ is called a **UTR-cluster** if there exists a time representative $c \in UTR$ such that $c \in \Gamma$. If Γ is a UTR-cluster $c \in UTR$) is its time representative, we will denote this by $\Gamma(c)$. We denote by $Clan(\Gamma)$ the set of s-clans included in Γ.
Let $\Gamma = \Gamma(c)$ and $\Delta = \Delta(d)$ be UTR-clusters. We consider UTR-clusters as abstract time points and define the **time order** between UTR-clusters as follows:
$\Gamma \prec \Delta$ iff $c\mathcal{B}d$.
We denote by **now** the UTR-cluster containing **NOW**.

Outline

The aim of the representation theory for DCA-s

• is to show that each DCA is isomorphic to a certain standard dynamic contact algebra. This means that if we have at hand a DCA $\underline{B}$,

• we, first, have to extract from $\underline{B}$ a time structure $(T, \prec, \mathbf{now})$;

• second, for each $m \in T$ to define in some way a contact algebra $\underline{B}_m$ (coordinate contact algebra of the time point m);

• and third, to define a standard DCA $\mathbf{B}^{can}$ (called *canonical DCA*) determined by the time structure $(T, \prec, \mathbf{now})$ and by the set of coordinate contact algebras $(\underline{B}_t, C_t)$, $t \in T$.

• The last step is to define an embedding h from $\underline{B}$ into the constructed standard dynamic contact algebra $\mathbf{B}^{can}$.

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• The last step is to define an embedding h from $\underline{B}$ into the constructed standard dynamic contact algebra $\mathbf{B}^{can}$.

Extracting the time structure

Definition

Formal definition of canonical time structure. Let $\underline{B}$ be a DCA. The canonical time structure $T(\underline{B})^{can} = (T, \prec, \mathbf{now})$ of $\underline{B}$ is defined as follows: T is the set of UTR-clusters of $\underline{B}$, the relation $\prec$ is the time order between UTR-clusters and **now** is the unique UTR-cluster containing **NOW**.

The UTR-clusters as elements of the canonical time structure are considered as the abstract time points of $\underline{B}$, **now** as the time point of **present epoch** and NOW is just the universal time representative of **now**.

Lemma

Correspondence Lemma. *Let $\underline{B}$ be a DCA and $T(B)^{can} = (T, \prec, \textbf{now})$ be the canonical time structure of $\underline{B}$. Let A be any formula from the list of* ***time conditions*** *(RS), (LS), (Up Dir), (Down Dir), (Dens), (Ref), (Irr), (Lin), (Tri), (Tr)*
and α be the corresponding formula from the list of ***time axioms*** *(rs), (ls), (up dir), (down dir), (dens), (ref), (irr), (lin), (tri), (tr).*
Then:
α is true in $\underline{B}$ iff A is true in $T(B) = (T, \prec, \textbf{now})$

Factor contact algebra by a set of clans.

Definition

Let $\underline{B}$ be a contact algebra and $\alpha \subseteq CLANS(\underline{B})$, $\alpha \neq \varnothing$. We construct a contact algebra (B_α, C_α) corresponding to α as follows:

- Define $I(\alpha) = \{a \in B : \alpha \cap h(a) = \varnothing\}$. $I(\alpha)$ is a proper ideal in $\underline{B}$, i.e. $1 \notin I(\alpha)$.
- The congruence defined by $I(\alpha)$ is denoted by $\equiv_\alpha$. The congruence class determined by $a \in B$ is denoted by $|a|_\alpha$.
- Define $\underline{B}_\alpha$ to be the factor a algebra $\underline{B}/\equiv_\alpha = B/I(\alpha)$. We define a contact relation C_α in $\underline{B}_\alpha$ as follows: $|a|_\alpha C_\alpha |b|_\alpha$ iff $\alpha \cap h(a) \cap h(b) \neq \varnothing$.

We apply this construction in the next slide

Extracting the standard DCA

• Let $\underline{B}$ be a DCA and Γ be a time point in $\underline{B}$, i.e. Γ is an UTR-cluster and let $Clans(\Gamma)$ be the set of s-clans included in Γ.

• First we construct the factor contact algebra $(\underline{B}_{Clans(\Gamma)}, C_{Clans(\Gamma)})$ by the set $Clans(\Gamma)$ with respect to the contact relation C^s.

• The algebra $(\underline{B}_{Clans(\Gamma)}, C_{Clans(\Gamma)})$ is called the **canonical coordinate contact algebra corresponding to the time point Γ.** Recall that the set Clans(Γ) determines a congruence relation in B and the elements of $\underline{B}_{Clans(\Gamma)}$ are just the equivalence classes $|a|_{Clans(\Gamma)}$ determined by this congruence relation. For simplicity of notation we will write C_Γ instead of $C_{Clans(\Gamma)}$ and similarly for $\underline{B}_\Gamma$, and $|a|_\Gamma$.

Coordinate contact algebras, canonical dynamic model of space and the embedding.

• Next, define the full standard DCA, denoted by $\underline{\mathbf{B}}^{can}$ with C^s, C^t and $\mathcal{B}$, TR, UTR, NOW in it as in Dynamic model of Space by means of the canonical time structure $T(B)^{can} = (T, \prec, \mathbf{now})$, and by the canonical coordinate algebras $(\underline{B}_\Gamma, C_\Gamma)$, $\Gamma \in T$.

• $\underline{\mathbf{B}}^{can}$ is called **full canonical standard DCA** corresponding to $\underline{B}$.

• The **canonical embedding** h is defined coordinatewise as follows: for each $a \in B$ and $\Gamma \in T$, $h(a)_\Gamma = |a|_\Gamma$.

Lemma

Embedding Lemma. *h is an embedding from $\underline{B}$ into the canonical full standard DCA $\underline{\mathbf{B}}^{can}$*

Theorem

Representation Theorem for DCA-s.
Let $\underline{B}$ be a DCA. Then there exists a full standard DCA $\mathbb{B}$ and an isomorphic embedding h of $\underline{B}$ into $\mathbb{B}$. Moreover, $\underline{B}$ satisfies some of the time axioms iff the same axioms are satisfied in $\mathbb{B}$.

Proof.

The proof is a direct corollary of the Embedding Lemma and the Correspondence Lemma by taking $\mathbb{B} = \underline{\mathbf{B}}^{can}$. □

The present paper is a third one in the series:

• D. Vakarelov, Dynamic Mereotopology: A point-free Theory of Changing Regions. I. Stable and unstable mereotopological relations. *Fundamenta Informaticae*, (2010)

• D. Vakarelov, Dynamic mereotopology II: Axiomatizing some Whiteheadian type space-time logics. In: *Advances in Modal Logic - 2012*.

Comments

Some **other works** related to our approach are

• Ivo Düntsch and M. Winter. Moving Spaces. In TIME-2008,

• I. Düntsch and M. Winter, Timed Contact Algebras. In: TIME-2009.

They are point-free with respect to space points but not with respect to time points in the sense that the set of time points is explicitly given in the axiomatization.

Modal logics for **Minkowski space-time**, based on different ideas, are considered in

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T H A N K S