

NON-CLASSICAL LOGICS

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A. Heyting, A.N. Kolmogorov, V.I. Glivenko

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NOVOSIBIRSK STATE UNIVERSITY (1960 – 1965)

SOBOLEV INSTITUTE OF MATHEMATICS (1964 – now)

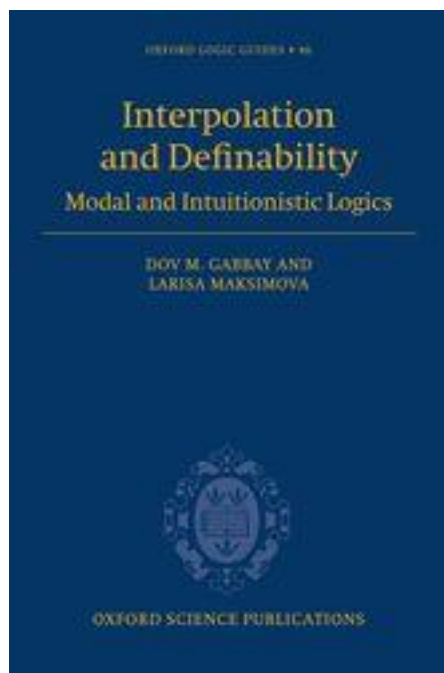
H. Rasiowa, R.Sikorski. The Mathematics of Metamathematics. Warszawa, 1963.



Helena RASOWA

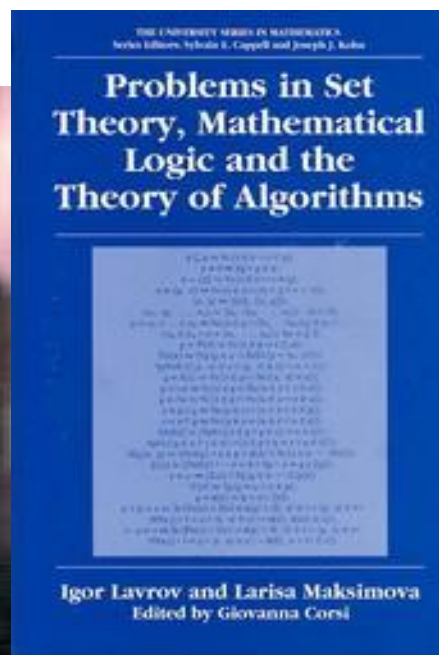
Ph D Thesis: Logical calculi of rigorous implication (1968)

Hab. Thesis: Decidable properties of superintuitionistic and modal logics (1985)



Dov Gabbay and L.Maksimova. Oxford University Press 2005

NOVOSIBIRSK STATE UNIVERSITY (from 1965)



I.A.Lavrov and L.L.Maksimova. Moscow, Nauka, 1st Ed. – 1975, 5th – 2002-06;

Hungarian transl. – Budapest, 1987; Polish – Warsaw, 2004;

English - New York, Kluwer 2002.

MAIN RESULTS (1963 – 2013)

RELEVANT LOGICS

- **Algebraic semantics for relevant logics, representation theorems**
- **Relational semantics for entailment**

SUPERINTUITIONISTIC AND MODAL LOGICS

• There are exactly three pretabular s.i. logics and exactly five pretabular modal logics over S4. Tabularity problems over Int and S4 are decidable (NP-complete). (1972, 1975)

• Tabularity problems for varieties of Heyting algebras and of interior algebras are base-decidable

INTERPOLATION AND AMALGAMATION

CIP: If A implies B then there is a C in the common language such that A implies C and C implies B .

W.Craig 1957

- **There are exactly eight s.i. logics with CIP. All of them are recognizable (1977).**
- **There are not more than 37 modal logics over S4 with CIP. All of them are recognizable. CIP is decidable over Int and S4 (1979).**
- **Interpolation = Amalgamation.**

INTERPOLATION AND BETH DEFINABILITY

Theorem (E. Beth, 1953). If a predicate is implicitly definable in a first-order theory, then it is explicitly definable.

PBP: If $A(P, Q, x)$, $A(P, Q', y)$ derive $(x \leftrightarrow y)$, then there is a $B(P)$ such that $A(P, Q, x)$ derives $(x \leftrightarrow B(P))$.

BP: Omit Q in PBP.

• **Temporal logics with linear time and with branching time have neither CIP nor BP (1991-92)**

• **All extensions of the modal K4 logic have BP (1992)**

Variants of interpolation and of Beth definability:

CIP, IPD, IPR, WIP; PBP, PB1, BP, B1.

Their equivalents: different variants of amalgamation and epimorphisms surjectivity.

- **All s.i.l. and positive logics with these properties are described. All these properties are decidable (2000-2010).**
- **All these properties are decidable over modal S4 logic. There are only finitely many logics with PBP and IPR over S4. All of them are recognizable over S4 (2013).**
- **WIP is decidable over the Johansson minimal logic (2011).**

THANK YOU!