Atomic Rule Refinement in the Tableau Synthesis Framework

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MAL'TSEV MEETING

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Outline

Prover Development Problem

- Different applications require different (logical) formalisms
- Logics need reasoning tools
- Implementation of provers is expensive
- Altering existing provers is hard
- Translational approach requires additional knowledge and skills for the user

Solution

Generation of a prover code from a specification of a logic.

Tableau-Based Reasoning

- Has a long tradition and is a well established method in automated reasoning
- Approach can be successfully used for a large number of logics:
  - Boolean logic, first-order logic, higher-order logics,
  - modal, description, hybrid, superintuitionistic logics, . . . ,
  - temporal, dynamic, fix-point logics, . . .
- Multitude of different tableau approaches:
  - ground semantic tableau
  - free-variable tableau,
  - hypertableau, . . .
- Many implemented systems

Automation of Calculus Development

- Existing work for non-classical logics suggests that it should be possible to develop tableau calculi systematically for large classes of logics
  - many variations
  - many similarities
  - important underlying principles tend to be the same
- Questions:
  - Can tableau calculi be developed automatically from the definition of logics?
  - Can soundness and completeness be guaranteed?
  - Can termination be guaranteed?
- No, in each case.
- Is it possible to develop tableau calculi automatically under certain restrictions?

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Tableau Calculus Synthesis Framework

- A method of synthesis of a sound and complete tableau calculus from a first-order semantic specification of a logic.
- Calculus refinement methods.
- A generic blocking mechanism which ensures termination of tableau algorithms for logics with the finite model property.

Refinement Methods

- Simplification of the tableau language
-

Refinement Methods

- Simplification of the tableau language
- Our focus: Reducing branching factor of the tableau rules

Tableau Language = Language of Semantics

- A multi-sorted first-order language (extending the language of given logic)
- Connectives of the logic = functional symbols
- Formulae of the logic = terms of the tableau language
- Truth predicates ν_s for each sort s of the logic

$$M \models \nu_{\mathsf{f}}(\phi, a) \;\equiv\; M, a \models \phi$$

- Equality predicates $\approx$ for each sort.

Basic Notions

- Tableau rule:

$$\frac{X}{X_1 \mid \cdots \mid X_m}$$

- Tableau as a search space
- Tableau branch
- Soundness
- Completeness
- Termination

Tableau Synthesis

- Express semantics S of a logic
- Transform semantics to a ‘well-defined’ form
- Eliminate quantifiers by Skolemisation
- Generate tableau calculus T_S from the transformed semantics

Theorem (Soundness)

T_S is sound for the logic specified by S, i.e.

if a formula ϕ is satisfiable in an S-model then
every tableau derivation for ϕ based on T_S contains an open branch.

Generating Tableau Calculus for S4

$$\forall x(\nu(\neg p, x) \leftrightarrow \neg\nu(p, x))$$
$$\forall x(\nu(p \vee q, x) \leftrightarrow \nu(p, x) \vee \nu(q, x))$$
$$\forall x(\nu(\Diamond p, x) \leftrightarrow \exists y(R(x, y) \wedge \nu(p, y)))$$
$$\forall x, y, z(R(x, y) \wedge R(y, z) \rightarrow R(x, z))$$
$$\forall x R(x, x)$$

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$$\forall x(\nu(\Diamond p, x) \to (R(x, f(p, x)) \wedge \nu(p, f(p, x))))$$

$$\forall x, y(\neg\nu(\Diamond p, x) \to (\neg R(x, y) \vee \neg\nu(p, y)))$$

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$$\frac{\nu(p, x), \ \neg\nu(p, x)}{\bot} \qquad \frac{R(x, y), \ \neg R(x, y)}{\bot}$$

+equality rules

Constructive Completeness

Given open branch $\mathcal{B}$ in a derivation in T_S, build a model

$$\mathcal{I}(\mathcal{B}) \stackrel{\text{def}}{=} \langle \Delta^{\mathcal{I}(\mathcal{B})}, P^{\mathcal{I}(\mathcal{B})}, \ldots, \nu_{\mathsf{s}}^{\mathcal{I}(\mathcal{B})}, \ldots \rangle$$

- $\|t\| \stackrel{\text{def}}{=} \{t' \mid t \approx t' \text{ is in } \mathcal{B}\}$
- $\Delta^{\mathcal{I}(\mathcal{B})} \stackrel{\text{def}}{=} \{\|t\| \mid t \text{ is in } \mathcal{B}\}$
- $(p, \overline{\|t\|}) \in \nu_{\mathsf{s}}^{\mathcal{I}(\mathcal{B})} \stackrel{\text{def}}{\iff} \nu_{\mathsf{s}}(p, \bar{t}) \in \mathcal{B}$
- $\overline{\|t\|} \in P^{\mathcal{I}(\mathcal{B})} \stackrel{\text{def}}{\iff} P(\bar{t}) \in \mathcal{B}$

T_S is *constructively complete* iff (for every such $\mathcal{B}$)
$\mathcal{I}(\mathcal{B})$ is extendable to an S-model which *reflects* $\mathcal{B}$.[1]

Constructive Completeness $\Longrightarrow$ Completeness.

Theorem (Constructive Completeness)

(If S is a well-defined semantic specification then)
T_S is constructively complete.

[1] i.e. validates all the literals in $\mathcal{B}$ under the valuation $t \mapsto \|t\|$.

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General Rule Refinement

- Let $X_1 = \{\psi_1, \dots, \psi_k\}$ and

$$r \stackrel{\text{def}}{=} \frac{X}{X_1 \mid \cdots \mid X_m} \qquad r_j \stackrel{\text{def}}{=} \frac{X \cup \{\sim\psi_j\}^1}{X_2 \mid \cdots \mid X_m}$$

- $T'_S \stackrel{\text{def}}{=} (T_S \setminus \{r\}) \cup \{r_1, \dots, r_k\}$.
- For every open branch $\mathcal{B}$ in a T'_S-tableau, if $E_1, \dots, E_l$ are reflected in $\mathcal{I}(\mathcal{B})$ then

(†) $X(\overline{E}, \overline{t}) \subseteq \mathcal{B}$ implies $\mathcal{I}(\mathcal{B}) \models X_i(\overline{E}, \overline{\|t\|})$ for some $i = 1, \dots, m$.

Theorem (Refinement)

T'_S is sound and constructively complete whenever T_S is.

[1] $\sim\phi \stackrel{\text{def}}{=} \psi$ if $\phi = \neg\psi$ and $\sim\phi \stackrel{\text{def}}{=} \neg\phi$ if $\phi \neq \neg\psi$

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Atomic Rule Refinement

- $\mathcal{L}$-*atomic* formula = atomic formula in *FO* ($\mathcal{L}$) which does not contain complex $\mathcal{L}$-terms:

$$\nu_{\mathsf{f}}(p,t) \qquad R(t_1,t_2) \qquad \nu_{\mathsf{r}}(r,t_1,t_2,t_3)$$

- For every open branch $\mathcal{B}$ in a T'_S-tableau, if $E_1,\ldots,E_l$ are reflected in $\mathcal{I}(\mathcal{B})$ then

$X_0(\overline{E},\bar{t}) \subseteq \mathcal{B}$ implies

$X_1(\overline{E},\bar{t}) = \{\neg\xi_1,\ldots,\neg\xi_k\}$ and all $\xi_1,\ldots,\xi_k$ are $\mathcal{L}$-*atomic*.

Theorem (Atomic Refinement)

T'_S is sound and constructively complete whenever T_S is.

Box Rule

$$\frac{\neg\nu(\Diamond p, x)}{\neg R(x,y) \mid \neg\nu(p,y)}$$

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For more expressive modal-like logics the rule

$$\frac{\neg\nu_{\mathsf{f}}(\Diamond p, x)}{\neg\nu_{\mathsf{r}}(r,x,y) \mid \neg\nu_{\mathsf{f}}(p,y)}$$

may not be refinable, e.g. tableaux for logics with the relation complement.

Frame Conditions

$$\frac{}{\neg R(x,y) \mid \neg R(y,z) \mid R(x,z)}$$

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$$\overline{\neg R(x,y) \mid \neg R(y,z) \mid R(x,z)}$$

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Hypertableau Rule

Assumptions:

- The tableau for the logic contains rules:

$$\frac{\nu_{\mathsf{s}}(\neg p, \overline{x})}{\neg \nu_{\mathsf{s}}(p, \overline{x})} \quad \text{and} \quad \frac{\nu_{\mathsf{s}}(p \vee q, \overline{x})}{\nu_{\mathsf{s}}(p, \overline{x}) \mid \nu_{\mathsf{s}}(q, \overline{x})}$$

- AC for disjunction (for simplicity)

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$$\frac{\nu_{\mathsf{s}}(p_1 \vee \cdots \vee p_k, \overline{x})}{\nu_{\mathsf{s}}(p_1, \overline{x}) \mid \cdots \mid \nu_{\mathsf{s}}(p_k, \overline{x})} (k > 1)$$

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($m + n > 1$ and only atomic substitutions are allowed into $p_1, \ldots, p_m$)

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($m + n > 1$ and only atomic substitutions are allowed into $p_1, \ldots, p_m$)

Hypertableau
Algorithm

- Apply rules with higher branching factor less often.
- For every instance of the hypertableau rule

$$\frac{\nu_{\mathsf{s}}(\neg p_1 \vee \cdots \vee \neg p_m \vee \phi_1 \vee \cdots \vee \phi_n, \overline{x}), \quad \nu_{\mathsf{s}}(p_1, \overline{x}), \quad \cdots, \quad \nu_{\mathsf{s}}(p_m, \overline{x})}{\nu_{\mathsf{s}}(\phi_1, \overline{x}) \mid \cdots \mid \nu_{\mathsf{s}}(\phi_n, \overline{x})},$$

decompose $\nu_{\mathsf{s}}(\phi_i, \overline{x})$ by tableau rules.

Hypertableau
Improvement

- Assume that for every ϕ from a large subclass of formulae of given logic:

$$\nu_{\mathsf{s}}(\phi, \overline{x}) \leftrightarrow \bigwedge_{i=1}^{I} \Big(\nu_{\mathsf{s}_{ij}}(\bigvee_{j=1}^{J_i} \phi_{ij}, \overline{x}) \vee \bigvee_{k=1}^{K_i} L_{ik} \Big).$$

- Assume that there is an *efficient* reduction algorithm $\mathcal{A}$ for every such $\nu_{\mathsf{s}}(\phi, \overline{x})$ to its equivalent clausal form.
- Replace every new $\nu_{\mathsf{s}}(\phi, \overline{x})$ in the derivation by its equivalent clausal form.

Hypertableau for S4

$$\frac{\nu(\neg p, x)}{\neg\nu(p, x))} \quad \frac{\neg\nu(\neg p, x)}{\nu(p, x)}$$

$$\frac{\nu(p \vee q, x)}{\nu(p, x) \mid \nu(q, x)} \quad \frac{\neg\nu(p \vee q, x)}{\neg\nu(p, x), \quad \neg\nu(q, x)}$$

$$\frac{\nu(\Diamond p, x)}{R(x, f(p, x)), \quad \nu(p, f(p, x))} \quad \frac{\neg\nu(\Diamond p, x), \quad R(x, y)}{\neg\nu(p, y)}$$

$$\frac{R(x, y), \quad R(y, z)}{R(x, z)}$$

$$\frac{}{R(x, x)}$$

$$\frac{\nu(p, x), \quad \neg\nu(p, x)}{\bot}$$

+equality rules

Hypertableau for S4

$$\frac{\nu(\neg p, x)}{\neg\nu(p, x))} \qquad \frac{\neg\nu(\neg p, x)}{\nu(p, x)}$$

$$\frac{\nu(\neg p_1 \vee \cdots \vee \neg p_m \vee q_1 \vee \cdots \vee q_n, x), \ \ \nu(p_1, x), \ \ \cdots, \ \ \nu(p_m, x)}{\nu(q_1, x) \ \mid \ \cdots \ \mid \ \nu(q_n, x)}$$

($m + n > 1$ and only atomic substitutions are allowed into $p_1, \ldots, p_m$)

$$\frac{\neg\nu(p \vee q, x)}{\neg\nu(p, x), \ \ \neg\nu(q, x)}$$

$$\frac{\nu(\Diamond p, x)}{R(x, f(p, x)), \ \ \nu(p, f(p, x))} \qquad \frac{\neg\nu(\Diamond p, x), \ \ R(x, y)}{\neg\nu(p, y)}$$

$$\frac{R(x, y), \ \ R(y, z)}{R(x, z)}$$

$$\overline{R(x, x)}$$

$$\frac{\nu(p, x), \ \ \neg\nu(p, x)}{\bot}$$

+equality rules

Conclusion

- Description of a tableau synthesis framework
- A generic method for refinement of tableau rules
- An atomic rule refinement condition to guarantee soundness and completeness
- Refinement of box rule and frame conditions in modal-like logics
- Application the atomic rule refinement method in order to obtain a hypertableau-like calculus

Thank You! Questions?

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