

SOME MODAL OPERATORS OVER INTUITIONISTIC LOGIC

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We study a number of modal operators over intuitionistic logic defined as compositions of principal modal operator of logic in consideration with intuitionistic negation. We adopt an approach due to K. Došen and M. Božić (see [1] and [3]), who introduced four systems of intuitionistic modal logics — $HK\Box$, $HK\Diamond$, $HK\Box'$ and $HK\Diamond'$ — each dealing with one of the modal operators of necessity, possibility, un-necessity and impossibility, respectively, as a framework for investigating intuitionistic modal logics. By a composition we understand simply a sequence consisting of occurrences of one of four types of modal operators and intuitionistic negation and we mainly focus on basic compositions, that is compositions of the form $\neg\delta$, $\delta\neg$ or $\neg\delta\neg$, where δ is the principal modal operator of the logic $HK\delta$. It is well known that such compositions over classical logic can be regarded as natural definitions of four types of modal operators above by means of each other.

We investigate which basic compositions yield modal operators of the same type over intuitionistic logic as over classical logic, which is a natural question, as there is no dualities between four types of modal operators over intuitionistic logic. It turns out that five basic compositions behave classically in that sense and for those five compositions we obtain their respective axiomatizations. Moreover, we show that logic KC is the smallest superintuitionistic logic over which all twelve basic compositions behave classically.

We also investigate the so called Heyting-Kripke logic N^* [2], which can be obtained by adding to intuitionistic non-modal base modal operator \sim , satisfying exactly the properties of negative operators \Box' and \Diamond' in logics $HK\Box'$ and $HK\Diamond'$, respectively. We study some basic properties of N^* such as finite model property, decidability and constructive properties. We then axiomatize two compositions, that is $\neg\sim$ and $\sim\sim$, in N^* as necessity operators. We show that the former composition has an infinite axiomatization and the latter one is axiomatized in the form of logic, modal operator in which combines properties of positive operators \Box and \Diamond in logics $HK\Box$ and $HK\Diamond$, respectively.

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