ASYMPTOTIC DENSITY AND THE THEORY OF COMPUTABILITY

In recent years the asymptotic-generic point of view of geometric group theory has led to new developments in the theory of computability. A set A is *generically computable* if there is a partial algorithm ϕ for A whose domain has asymptotic density 1. A set A is *coarsely computable* if there is a computable set C such that the symmetric difference of A and C has density 0. Natural questions from this point of view turn out to be very closely linked to classical ideas in computability theory. For example, a c.e. degree $\boldsymbol{d}$ is *not* low if and only if $\boldsymbol{d}$ contains a c.e. set A with density 1 which does not have any computable subset of density 1.

It turns out that there is a very tight connection between the position of a set in the arithmetic hierarchy and the complexities of its densities as real numbers. For example, a real number r is left-Π_2 if and only if it is the density of a c.e. set. For each $n \geq 2$, a real number r is the difference of two left-Π_2 reals if and only if it is the density of an n-c.e. set. We also discuss computability at densities less than 1.