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The filter of supergeneric sets

This talk presents a paper that was published last year in the *Journal of Algebra 404, p. 240-270,* under the title *Supergénérix.*

Its starting point was the following problem : given a linear group G , embedded in a matrix group Glm(K) , what can be said on the trace on G of a constructible subset C (i.e. a boolean combination of a finite number of Zariski-closed sets) of Glm(K) ? We observe that there is a subgroup H of finite index in G such that, for each coset aH , the intersection $C \cap aH$ is a translate of a supergeneric or of a cosupergeneric subset of H .

Given an arbitrary group G and a subset A of it, we say that A is *generic* if a finite number of translates of A cover G : $G = a_1A \cup ... \cup a_mA$; we says that it is *supergeneric* if any intersection $B = b_1A \cap ... \cap b_nA$ of a finite number of translates of A is generic (there are three notions : left, rigth , two-sided generic and supergeneric). A basic fact is that the supergeneric sets form a filter, the intersection of two supergeneric sets being also supergeneric.

We shall consider some uniformity properties possessed by supergeneric sets which are the traces of the definable subsets of an overgroup Γ of G having nice model-theoric properties (this generalizes the linear case), and also the behaviour of supergenericity with respect to cartesian products in this context. We shall also illustrate the notion with very plain groups, such that the infinite cyclic group and the Prüfer groups, and conclude by a list of open questions.