BASES AND GENERATING SETS IN COMPUTABLE ALGEBRA

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We will survey several recent results in computable algebra that rely on various notions of independence and different choices of generating sets as technics tools. These results can be roughly sub-divided into two kinds.

Results of the first kind study classes in which the respective notions of independence and bases are natural, and it makes sense to study these notions on their own right. Results of this kind go back to the Novosibirsk School of Algebra and Logic and to Mal'cev in particular. As was first noted by Goncharov, results of this sort have interesting applications in the theory of computable categoricity and auto-dimension. Using the notion of a c.e. pregeometry, we develop a general framework of independence in computable commutative structures with new applications to computable differentially closed, difference closed, and real closed fields. We also discuss a long-standing open problem of tree-bases in abelian p-groups.

Results in the second class use special notions of a generating set as a technical tool. With a right choice of a generating set, many complicated arguments become a lot simpler. In fact, several known proofs of this sort would be totally incomprehensible in absence of a good choice of a generating set. Such results are common in computable abelian group theory, but there have been applications to other classes including integral domains.

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