# MONADIC SECOND-ORDER DEFINABILITY IN SOME WEAK ARITHMETICAL STRUCTURES

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This talk surveys some results on monadic second-order definability in (relatively) weak arithmetical structures. Perhaps the most important of these structures are

$$\langle \mathbb{N}; \leqslant \rangle, \quad \langle \mathbb{N}; +, = \rangle, \quad \langle \mathbb{N}; \times, = \rangle, \quad \langle \mathbb{N}; \mid \rangle \quad \text{and} \quad \langle \mathbb{N}; \perp \rangle$$

where $\mid$ and $\perp$ denote the divisibility relation and the coprimeness relation, respectively — i.e., for any $\{n, k\} \subseteq \mathbb{N}$, we have

$$\begin{aligned} n \mid m \quad &\iff \quad n \text{ divides } m, \quad \text{and} \\ n \perp m \quad &\iff \quad n \text{ and } m \text{ have no common prime divisor.} \end{aligned}$$

I shall pay special attention to them in my talk. Also, I shall mention some related results on first-order definability (in particular, those by A. Bès, P. Cegielski, Yu. V. Matiyasevich, D. Richard, J. Robinson and A. R. Woods). See the references to this abstract for further details.

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## References

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