FROM LOCAL TO GLOBAL CONJUGACY IN RELATIVELY HYPERBOLIC GROUPS

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Relatively hyperbolic groups were introduced by Gromov (1987). It is known (see Hrushka (2010)) that essentially different definitions given by Gromov, Bowditch, and Osin are coincide in the case where the ambient group is countable and the peripheral subgroups are infinite. In the first part of my talk I will recall the combinatorial definition of Osin and formulate some statements which help to work in this area. In the second part I will explain new results concerning the conjugacy of subgroups in relatively hyperbolic groups. In particular, we prove the following theorem:

Theorem. *Suppose that a finitely generated group G is hyperbolic relative to a collection of subgroups P. Let H_1 and H_2 be subgroups of G such that H_1 is relatively quasiconvex with respect to P and H_2 has a nonparabolic element of infinite order. Suppose that H_2 is elementwise conjugate into H_1. Then there exists a finite index subgroup of H_2 which is conjugate into H_1.*

The minimal length of the conjugator can be estimated.

It is known that limit groups (see Zela, Remeslennikov, Miasnikov, Kharlampovich) are hyperbolic relatively to representatives of conjugacy classes of maximal noncyclic abelian subgroups.

Corollary 1. *Let G be a limit group and let H_1 and H_2 be subgroups of G, where H_1 is finitely generated. Suppose that H_2 is elementwise conjugate into H_1. Then there exists a finite index subgroup of H_2 which is conjugate into H_1. The index depends only on H_1.*

The minimal length of the conjugator can be estimated.

Corollary 2. *Limit groups are subgroup conjugacy separable. In particular, the conjugacy problem for two finitely generated subgroups of a limit group is decidable.*

The last corollary was proved independently by Chagas and Zalesskii by other metods.

Other theorems concerning hyperbolic virtually compact special groups will be given in the talk (this class of groups appeared in the course of solution of the virtually Haken conjecture on 3-manifolds).

This is a joint work with K.-U. Bux

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