MEASURE AND CATEGORICITY FOR CLASSES OF COUNTABLE STRUCTURES

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We consider classes of countable structures for which the isomorphism problem is not overly difficult. Examples include the class of algebraic fields of characteristic 0, the class of finite-branching trees under the predecessor function, the class of torsion-free abelian groups of a fixed finite rank n, and the class of real closed fields of a fixed finite transcendence degree d. The isomorphism problems are Π^0_2 for the first two of these, Σ^0_3 for the abelian groups, and Π^0_3 for the real closed fields.

The first goal for such a class is to give an effective classification of all structures in the class, up to isomorphism. Such a classification is computed by two Turing functionals Φ and Ψ. For every atomic diagram D of an element of the class, the index $A = \Phi^D$ for its isomorphism type should be an element of a known topological space (usually Cantor space 2^ω or Baire space ω^ω, possibly modulo a standard equivalence relation), and for every A in this index space, $D = \Psi^A$ should be the atomic diagram of a structure of the isomorphism type mapped to A. Thus Φ and Ψ constitute a computable homeomorphism between the class, modulo isomorphism, and the index space.

In some cases, it is then possible to put a probability measure on the index space, and thus to ask about the measure of the subclass consisting of those structures satisfying a particular property. We will address this question in the case of the algebraic fields, for the property of uniform computable categoricity. Current joint work by the author and Johanna Franklin considers the same question for the finite-branching trees.

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