SYMMETRONS

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The aim of this contribution is to investigate the symmetric structure associated to a group of finite Morley Rank without involutions.

A protosymmetron is a structure S with a binary operation $s(x, y)$, in which, when we fix $y = a$, the unary operation $s(x, a)$ is called the symmetry with center a; it satisfies the equations stating that each symmetry is an involutive automorphism of the structure fixing its center. If moreover for all $x, x' \in S$ there is a unique y, called the middle of x and x', such that $s(x, y) = x'$, then S is called a symmetron; the symmetrons form a variety in the language of symmetry and middle.

Any group G with the operation $s(x, y) = y.x - 1.y$ is a protosymmetron, which is a symmetron when each point in G has a unique square root; this is the case of groups without involutions which are finite, algebraic, or more generally of finite Morley Rank.

Needless to say, two non-isomorphic groups may have isomorphic symmetrons.

The finite (and even locally finite) symmetrons are fully described by Glauberman's Theorem.

We extend to symmetrons of finite Morley Rank some properties that are well known for groups:

— for each definable subset X of S, to be closed under symmetry is the same thing as to be closed under middle;

— S is decomposed in a unique way as a finite disjoint union of definable subsymmetrons of Morley Degree one, which we call its connected components;

— two definable connected subsymmetrons of S with non-empty intersection generate (using symmetry and middle) in a bounded way a definable connected subsymmetron of S;

— the equivalence relation "there is a definable connected subsymmetron boundedly generated by X containing x and y" partitions the definable set X into a finite number of classes.

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