MULTIPLICATIVE-ADDITIVE LAMBEK CALCULUS: ITS COMPLEXITY AND ITS LANGUAGE AND RELATIONAL MODELS

A. SCEDROV

The Lambek calculus was introduced as a mathematical description of the syntax of natural languages. The Lambek calculus is a logical foundation of categorial grammar, a linguistic paradigm of grammar as logic and parsing as deduction. The calculus can also be viewed as a multiplicative fragment of non-commutative intuitionistic linear logic. In joint work with M. I. Kanovich and S. L. Kuznetsov, we consider the extension with the so-called additive connectives: conjunction and disjunction.

Language and relational models, or L-models and R-models, are two natural classes of models for the original Lambek calculus. Completeness w.r.t. L-models was proved by Pentus and completeness w.r.t. R-models by Andréka and Mikulás. It is well known that, because of the distributive law, adding both additive conjunction and disjunction together yields incompleteness. The product-free Lambek calculus enriched with conjunction only, however, is complete w.r.t. L-models (Buszkowski) as well as R-models (Andréka and Mikulás). The situation with disjunction turns out to be the opposite: we prove that the product-free Lambek calculus enriched with disjunction only is incomplete w.r.t. L-models as well as R-models.

The original Lambek calculus is NP-complete (Pentus), while its product-free fragment with only one implication is polynomially decidable (Savateev). It is known that the multiplicative-additive extension is PSPACE-complete (Kanovich, Kanazawa). In contrast with the polynomial-time result for the product-free Lambek calculus with one implication, we prove that the derivability problem is still PSPACE-complete even for a very small fragment $(\backslash, \wedge)$, including one implication and conjunction only. We also prove PSPACE-completeness for the $(\backslash, \vee)$ fragment, which includes only one implication and disjunction.

University of Pennsylvania, Philadelphia (USA)
Email address: scedrov@math.upenn.edu