

# DEZA GRAPHS: GENERALIZATION OF STRONGLY REGULAR GRAPHS

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A *Deza graph* is a simple regular graph for which the number of common neighbours of two different vertices takes just two values. Deza graphs were introduced by Erickson et al in [2], where the basics of Deza graph theory were founded and several new constructions were presented.

Deza graphs can be considered in terms of matrices. Suppose  $G$  is a graph with  $n$  vertices, and  $M$  is its adjacency matrix. Then  $G$  is a Deza graph with parameters  $(n, k, b, a)$  if and only if

$$M^2 = kI + aA + bB$$

for some symmetric  $(0, 1)$ -matrices  $A$  and  $B$  such that  $A + B + I = J$ , where  $J$  is the all ones matrix and  $I$  is the identity matrix.

If  $G$  is a Deza graph with  $M$ ,  $A$  and  $B$  as above, then  $A$  and  $B$  are adjacency matrices of graphs, and the corresponding graphs  $G_A$  and  $G_B$  are called the *children* of  $G$ .

Deza graphs whose children are complete multipartite graphs and their complement are known as *divisible design graphs*. Divisible design graphs were studied by W. H. Haemers, H. Kharaghani and M. Meulenberg in [3].

We call a Deza graph with strongly regular children as a *strongly Deza graph*. Obviously, divisible design graphs are strongly Deza graphs.

We study spectral properties of strongly Deza graphs and give some new constructions of divisible design graphs and strongly Deza graphs.

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