ALGEBRAIC AND COMBINATORIAL APPROACHES TO INVESTIGATING INTEGRAL GRAPHS

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In this talk we review recent progress in investigating integral graphs with emphasizing to new algebraic and combinatorial methods and techniques.

A graph is integral if all eigenvalues of its adjacency matrix are integers. Classification of integral graphs was started in 1974 by F. Harary but it is still uncompleted.

We develop dual Seidel switching techniques for getting new integral connected graphs [1]. The dual Seidel switching is an operation of a graph Γ interchanging only non-adjacent vertices by an order two automorphism of Γ. In particular, if Γ is integral then a graph obtained from Γ by the dual Seidel switching is integral as well. We obtained two new infinite families of integral graphs applying the dual Seidel switching to the Star graphs and to the Odd graphs. Moreover, we show that this method has a natural generalization for constructing strongly Deza graphs [5].

We also develop spectral and combinatorial approaches to obtain eigenfunctions with non-zero eigenvalues for the Star graphs being integral Cayley graphs over the symmetric group [2, 3]. A remarkable connection of new eigenfunctions known as PI-eigenfunctions with the standard basis of a Specht module of the symmetric group was established. Moreover, it was shown that any PI-eigenfunction with the eigenvalue $(n-2)$ is reconstructed by its values on the second neighbourhood of a vertex, and a basis of the eigenspace was found. The last result was used in [4] to show that the $(n-2)$-PI-eigenfunction has the minimum cardinality of the support.

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