METRIC SPACES AND COMPUTABILITY THEORY

ANDRÉ NIES

A function $f\colon \mathbb{N} \to \mathbb{N}$ is K-trivial if, up to an additive constant, each initial segment $f \upharpoonright n$ has the minimal complexity $K(n)$ in the sense of prefix-free Kolmogorov complexity K. Functions on $\mathbb{N}$ form points in Baire space with the ultrametric distance function. With Melnikov (Proc. AMS, 2013) we extended the concept of K-triviality to the setting of arbitrary computable metric spaces. Under some natural assumption on the metric space we proved that a K-trivial noncomputable point exists. We proved that a point is K-trivial if and only if is the limit of a fast converging sequence of basic points that is encoded by a K-trivial function. Very recently, the speaker together with Greenberg and Turetsky extended the theory of cost functions to the setting of computable metric spaces. This yields a dynamical characterization of K-trivial points via the speed of computable approximations.

We will also discuss work with A. Gavruskin (Lobachevskii Math. J., 2014) showing that among the metric spaces of diameter at most 1 with a distance function that is merely computably approximable from below, some object is universal for computable isometric embeddings. This is analogous to a result of the speaker with Ianovski et al. (J. Symb. Logic, 2014) that some Π^0_1 equivalence relation on $\mathbb{N}$ is universal with respect to computable many-one reductions.

The University of Auckland
E-mail address: andre@cs.auckland.ac.nz