ABOUT COMPLETE LATTICE HOMOMORPHIC IMAGES OF ALGEBRAIC LATTICES

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ABSTRACT. Algebraic lattices regarded as $(0, 1)$-lattices together with complete $(0, 1)$-lattice homomorphisms as morphisms form a category. We establish a duality result for this category. Using the duality we prove that the kernel of any complete lattice epimorphism f between algebraic lattices L and M is determined by a closed set S of some closure operator defined on the set of compact elements of L. It is determined by the following pattern

$$a \equiv b \ (\ker f) \Leftrightarrow S \ \cap \downarrow a = S \ \cap \downarrow b.$$

Here $\downarrow c$ means the set of all elements of L that are below c. Using this approach to complete congruences, we establish several results about lattices of quasivarietes and the complete homomorphic images of the ideal lattices, including the complete homomorphic images of the ideal lattices of free lattices. As the finite lattices are obviously algebraic, the above pattern shows that congruences on finite lattices behave in a way similar to the congruences on groups, rings, and modules, for example.

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