COMPARING THE COMPLEXITY OF IRRATIONAL NUMBER REPRESENTATIONS

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Irrational numbers can be represented in many different ways: Cauchy sequences, Dedekind cuts, continued fractions, etc. Without any restriction on the computations all known representations are equivalent: any given representation of an irrational number can be computably transformed into any other representation of the same number. But if we disallow unbounded search and consider subrecursive computations (for example, primitive recursive or elementary), this is no longer true. In fact, the structure of representations with respect to subrecursive reducibility becomes rather interesting and complex. As an example, in the following chain each representation (without the first) can be subrecursively converted into its neighbour on the left, but not conversely:

Cauchy sequence $<_{\mathcal{S}}$ base-b expansion $<_{\mathcal{S}}$ Dedekind cut $<_{\mathcal{S}}$ continued fraction.

My recent work explores the complexity of the Dedekind cut of some irrational numbers with specific patterns of zeros in their base-b expansion. In my paper [1] I give an example of an irrational number with elementary Dedekind cut and having regular long sequences of zero digits in its base-b expansion, but *after removing* these zeros the Dedekind cut of the obtained number can become arbitrarily complex. The other direction is still open: starting with an irrational number having elementary Dedekind cut, is it possible that the complexity of the Dedekind cut rises *after inserting* the same regular long sequences of zeros in the base-b expansion? In my talk I plan to present some new results, which are connected with this question. I will also mark future research directions: exploring new representations of irrational numbers and the adaption of the results to subclasses of the second Grzegorczyk class $\mathcal{E}^2$.

References

[1] Ivan Georgiev. Dedekind Cuts and Long Strings of Zeros in Base Expansions. *In: De Mol L., Weiermann A., Manea F., Fernández-Duque D. (eds) Connecting with Computability. Computability in Europe, 2021, LNCS, Springer, Cham*, vol. 12813 (2021), 248–259.

[2] Lars Kristiansen. On subrecursive representability of irrational numbers. *Computability*, vol. 6(3) (2017), 249–276.

[3] Lars Kristiansen. On subrecursive representability of irrational numbers, part II. *Computability*, vol. 8(1) (2019), 43–65.

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