SUBSTRUCTURAL LOGICS WITH KLEENE STAR

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In Gentzen-style sequent calculi, rules of inference break up into two groups: logical ones, which introduce logical connectives, and structural ones, operating the sequent structure. The latter include the rules of weakening, permutation, and contraction. Logical systems which lack all or some of those structural rules are called *substructural logics.* Propositional substructural logics are used for axiomatising atomic (equational) and Horn theories of *residuated lattices* and related classes of algebraic structures. A residuated lattice combines two structures sharing the same partial order: a lattice with its meet and join (also called additive conjunction and disjunction) and a monoidal operation (multiplicative conjuncton) with residuals w.r.t. the given partial order. Residuals are the substructural version of implication. In fact, if one identifies multiplicative and additive conjunction, a residuated lattices becomes a Heyting algebra. On the syntactic side, this restores structural rules and yields intuitionistic logic.

Natural examples of residuated lattices are algebras of formal languages and those of binary relations. Such algebras have an extra operation—iteration, or *Kleene star* (for binary relations, Kleene star is the reflexive-transitive closure). Residuated lattices with Kleene star are called *residuated Kleene lattices* or *action lattices;* the corresponding substructural logic is called *action logic.* Kleene star is quite an intriguing operation. There are two kind of axiomatisations for Kleene star: a weaker, inductive-style one, and a stronger infinitary one, via an ω-rule. The latter yields a system called *infinitary action logic.*

Action logic and infinitary action logic, both being undecidable, have different algorithmic complexity properties. Action logic is finitely axiomatisable, thus, for this logic undecidability means Σ^0_1-completeness. In contrast, infinitary action logic and its extensions (by finite sets of extra axioms and by so-called subexponential modalities) feature a range of interesting complexity level, from Π^0_1 to Π^1_1, with hyperarithmetical classes in between.

In the talk, we shall give a survey of algorithmic and semantical results for action logic and its variants. These results include well-known ones by V. Pratt, D. Kozen, W. Buszkowski, E. Palka, and others, as well as recent results obtained by the author, solely and in co-authorship with S. Speranski and T. Pshenitsyn.

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