Two problems of Ershov in the numberings theory

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We say that a surjective mapping $\nu : \omega \mapsto \mathcal{F}$ is a *computable numbering* of a family $\mathcal{F} \subseteq \Sigma_n^{-1}$ of sets of the Ershov hierarchy if

$$\{< x, m >: x \in \nu(m)\} \in \Sigma_n^{-1}.$$

The notion of reducibility for numberings is presupposed to be used in the talk. A Rogers semilattice $\mathcal{R}(\mathcal{F})$ stands for the set of equivalent classes of the computable numberings of $\mathcal{F}$ ordered by reduction of numberings.

Our goal is to discuss history and the current state of studying two problems of Yu.L. Ershov concerning the following invariants of the Rogers semilattices $\mathcal{R}(\mathcal{F})$ when $\mathcal{F} \subseteq \Sigma_n^{-1}$:

- cardinality of $\mathcal{R}(\mathcal{F})$,
- possible number of minimal elements in $\mathcal{R}(\mathcal{F})$.

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